"Full Coverage": Circle Theorems

This worksheet is designed to cover one question of each type seen in past papers, for each GCSE Higher Tier topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com/homework, logging on, *Practise* → *Past Papers/Worksheets* (or *Library* → *Past/Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Use the circle theorem "Angle at centre is twice angle at circumference."

[Edexcel IGCSE Nov2009-4H Q15ai]

A, B, C and D are points on a circle, centre O. Angle ABC = 58°

Calculate the size of angle AOC.

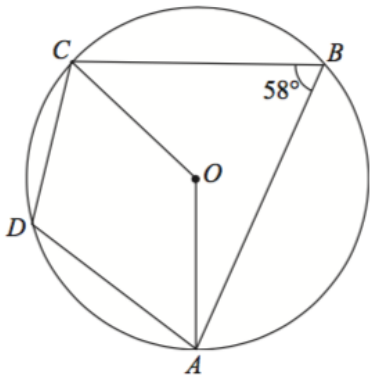

Diagram **NOT** accurately drawn

..... °

Question 2

Categorisation: Use this circle theorem combined with other basic angle properties, e.g. angles around a point add to 360°.

[Edexcel IGCSE Jan2017(R)-4H Q17a]

A, *B*, and *C* are points on the circumference of a circle, centre *O*.

DAE is a tangent to the circle.

Work out the size of angle *ACB*.

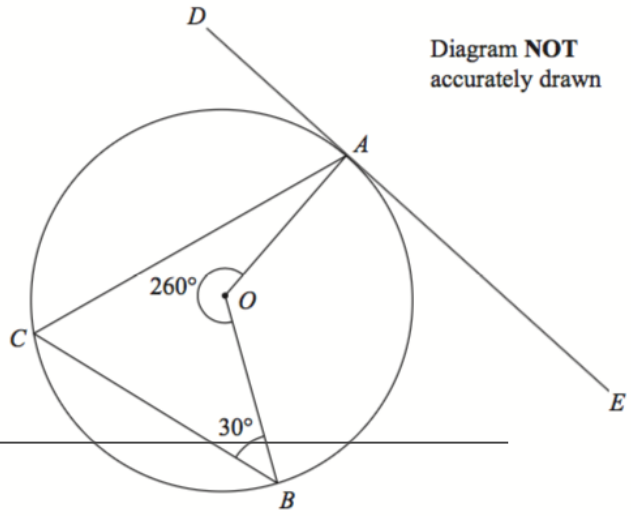

Diagram **NOT** accurately drawn

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Question 3

Categorisation: Be able to recite circle theorems.

[Edexcel IGCSE Jan2017-3H Q12bii Edited]

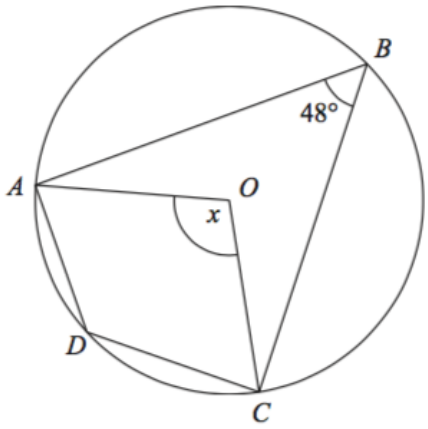

Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle with centre *O*. Angle *ABC* = 48°

Give a reason why angle *ADC*$= 132^{\circ}$.

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Question 4

Categorisation: Use the circle theorem "Opposite angles of a cyclic quadriteral add to 180°."

[Edexcel IGCSE June2011-3H Q16b]

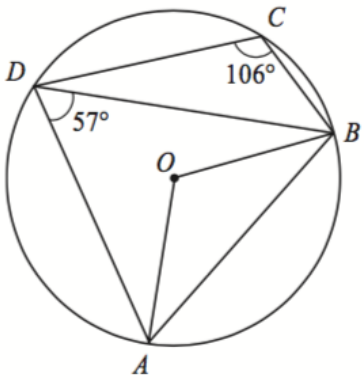

Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O. Angle ADB = 57° . Angle BCD = 106°

Calculate the size of angle BAD.

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Question 5

Categorisation: Use the circle theorems "Angle between radius and tangent is 90°".

[Edexcel GCSE June2012-1H Q21 Edited]

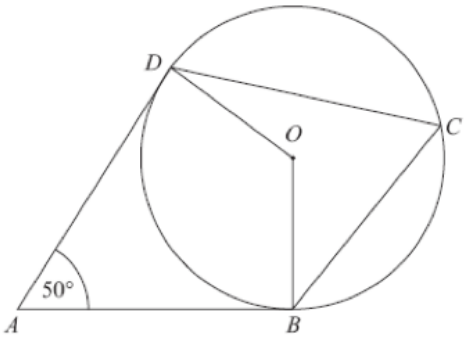

Diagram **NOT** accurately drawn

B, *C* and *D* are points on the circumference of a circle, centre *O*. *AB* and *AD* are tangents to the circle. Angle *DAB* = 50°
Work out the size of angle *BCD*.

angle $BCD =$ ..... °

Question 6

Categorisation: Recognise that the radius of a circle is of fixed length for a given circle.

[Edexcel GCSE March2012-3H Q19b Edited]

A, *B*, *C* and *D* are points on a circle, centre *O*.
BC = *CD*. Angle *BCD* = 130°
Work out the size of angle *ODC*.

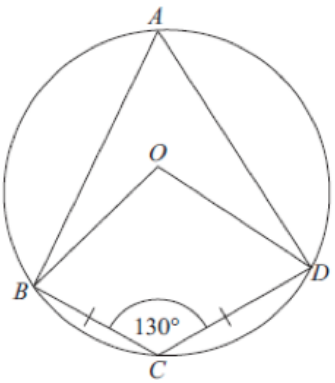

Diagram **NOT** accurately drawn

Angle ODC = ... °

Question 7

Categorisation: Use the circle theorem "Angles in the same segment are equal."

[Edexcel IGCSE Jan2012-3H Q13bi]

P, Q, R and S are points on the circumference of a circle.
PR and QS intersect at T. Angle QPR = 34° and angle PRS = 41°

Find the size of angle PTS.

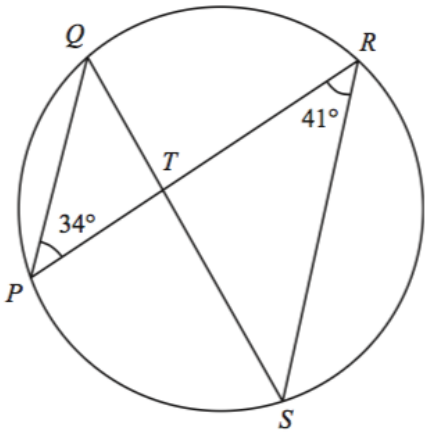

Diagram **NOT** accurately drawn

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Question 8

Categorisation: Use the circle theorem "Angle in a semicircle is 90°."

[Edexcel IGCSE May2014-4H Q16b]

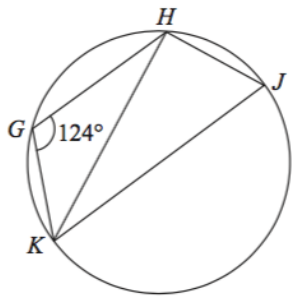

Diagram **NOT** accurately drawn

G, *H*, *J* and *K* are points on a circle. *KJ* is a diameter of the circle. Angle *KGH* = 124°
Work out the size of angle *HKJ*.

..... °

Question 9

Categorisation: Recognise that the "angle at centre is double angle at circumference" theorem still applies when the lines overlap.

[Edexcel GCSE June2009-3H Q18bii Edited]

The diagram shows a circle centre O. A, B and C are points on the circumference. DCO is a straight line. DA is a tangent to the circle.
Angle $ADO = 36^\circ$ and angle $AOD = 54^\circ$.
Determine angle ABC.

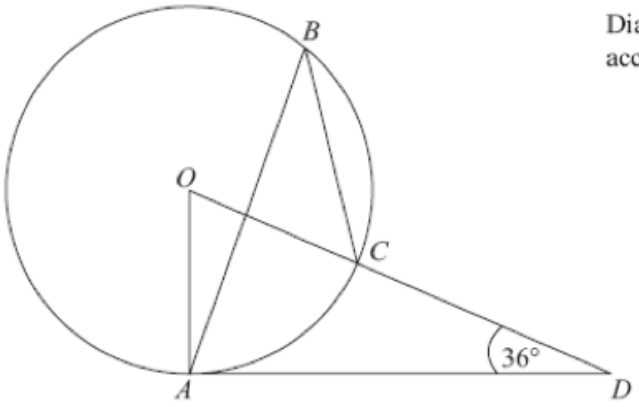

Diagram **NOT** accurately drawn

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Question 10

Categorisation: Use or recite the circle theorem "Tangents to a circle from a point are equal in length."

[Edexcel IGCSE May2013-4H Q19aii Edited]

The sides of triangle *PQR* are tangents to a circle.
The tangents touch the circle at the points *S*, *T* and *U*.
QS = 6 cm. *PS* = 7 cm.

Give a reason why $QT = 6$ cm.

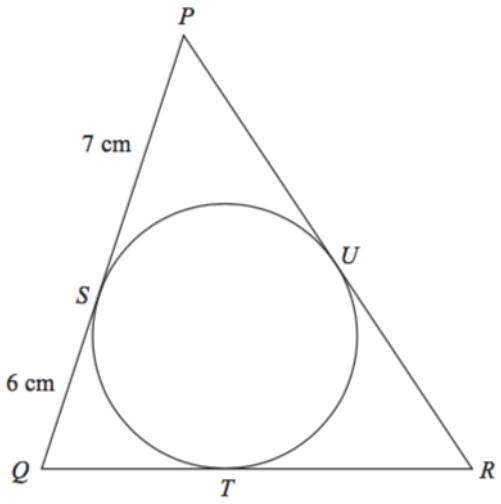

Diagram **NOT** accurately drawn

Question 11

Categorisation: Use the "Alternate Segment Theorem".

[Edexcel GCSE June2003-3I Q25ii, June2003-5H Q12ii]

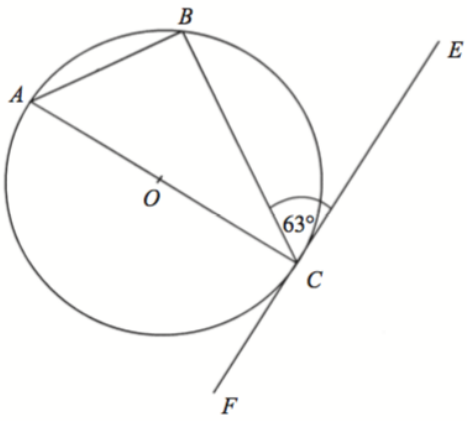

Diagram **NOT** accurately drawn

In the diagram, A, B and C are points on the circle, centre O.
Angle BCE = 63°
FE is a tangent to the circle at point C.

Calculate the size of angle BAC.
Give reasons for your answer.

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Question 12

Categorisation: Use circle theorems to form an equation and hence determine the value of a variable.

[Edexcel IGCSE May2012-3H Q18 Edited]

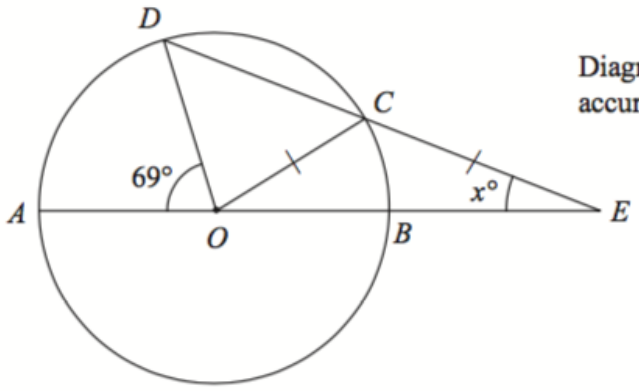


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*. *AOBE* and *DCE* are straight lines.
CO = *CE*. Angle *AOD* = 69° Angle *CEO* = x
Calculate the value of x .

..... °

Question 13

Categorisation: Use a mixture of circle theorems.

[Edexcel IGCSE June2010-3H Q13 Edited]

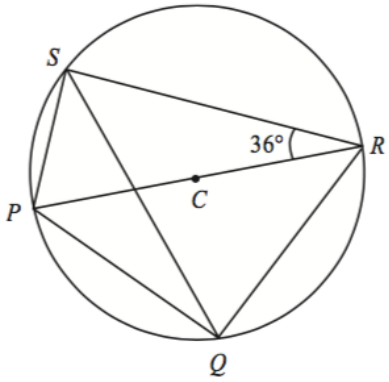


Diagram **NOT** accurately drawn

P, *Q*, *R* and *S* are points on a circle, centre *C*. *PCR* is a straight line.
Angle *PRS* = 36°
Calculate the size of angle *RQS*.

angle $RQS =$ ..... °

Question 14

Categorisation: Use a mixture of circle theorems.

[Edexcel IGCSE Jan2017(R)-4H Q17b]

A, *B*, and *C* are points on the circumference of a circle, centre *O*.
DAE is a tangent to the circle.

Work out the size of angle *CAD*.

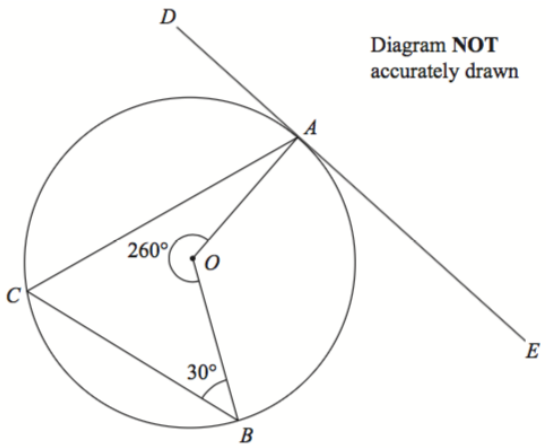


Diagram **NOT** accurately drawn

..... °

Question 15

Categorisation: Add lines to the diagram (typically the radius of the circle) to enable circle theorems to be used.

[Edexcel IGCSE Nov-2010-4H Q10]

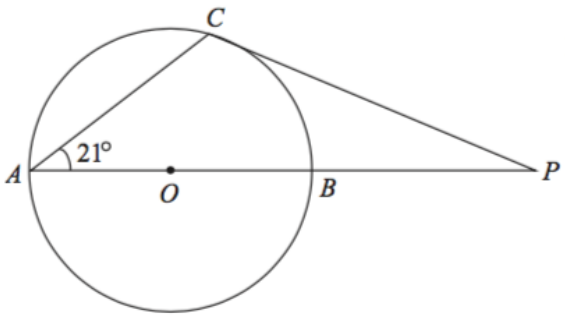


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O. AB is a diameter of the circle.
PC is a tangent to the circle. ABP is a straight line. Angle BAC = 21°

Work out the size of angle APC.

..... °

Question 16

Categorisation: Use a mixture of circle theorems.

[Edexcel IGCSE May2014(R)-4H Q18b]

ABCD is the tangent at *C* to a circle, centre *O.*
E, *F* and *G* are points on the circle.
AEOG and *BEF* are straight lines.

Angle *BAE* = 36°
Angle *EBC* = 70°

Find the size of angle *CGF*.

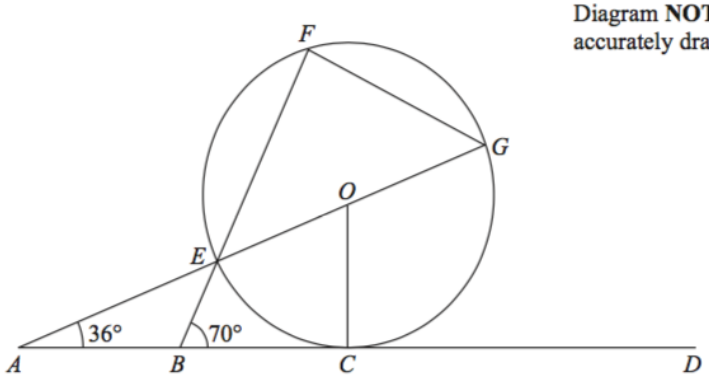


Diagram **NOT** accurately drawn

..... °

Question 17

Categorisation: Use a mixture of circle theorems.

[Edexcel IGCSE Jan2016-3H Q17b]

Q, R, S and T are points on a circle. ATB is the tangent to the circle at T

Angle $STR = 26^\circ$ Angle $RQT = 73^\circ$

Work out the size of angle STA

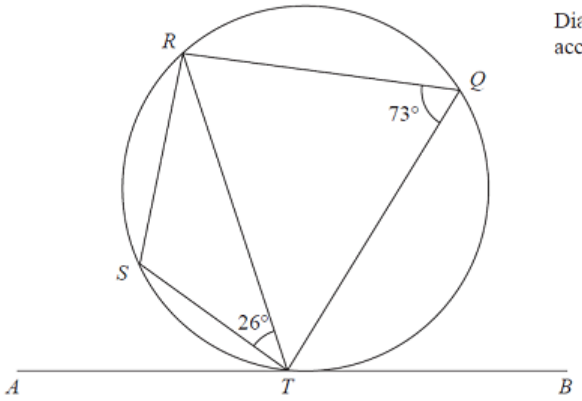


Diagram **NOT** accurately drawn

$\angle STA =$ ..... °

Question 18

Categorisation: Use a mixture of circle theorems.

[Edexcel GCSE Nov2014-2H Q17]

A, B, C and D are points on the circumference of a circle, centre O. AC is a diameter of the circle. AC and BD intersect at E .

Angle $CAB = 25^\circ$ Angle $DEC = 100^\circ$

Work out the size of angle DAC. You must show all your working.

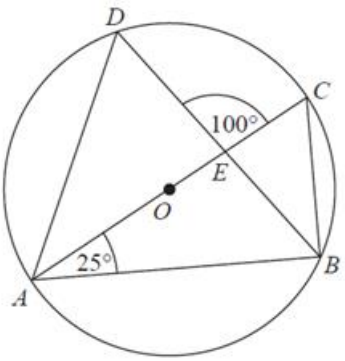


Diagram **NOT** accurately drawn

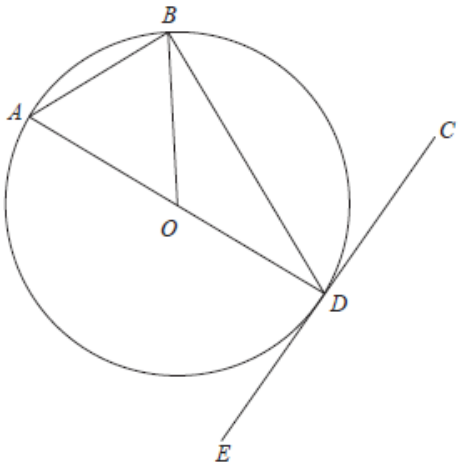
..... °

Question 19

Categorisation: Use a mixture of circle theorems.

[Edexcel GCSE(9-1) Mock Set 1 Autumn 2016 - 1H Q17]

A, B and D are points on the circumference of a circle centre O.
EDC is a tangent to the circle. Angle $BDC = 57^\circ$. Find the size of angle AOB.
You must give a reason for each stage of your working.



$\angle AOB =$ ..... $^\circ$

Question 20

Categorisation: Give an algebraic expression for an angle.

[Edexcel GCSE Nov2013-1H Q22]

A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
Angle *AOC* = y.
Find the size of angle *ABC* in terms of y.
Give a reason for each stage of your working.

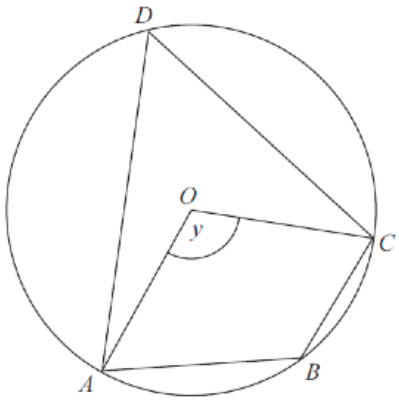


Diagram **NOT** accurately drawn

Angle $ABC =$ .....

Question 21

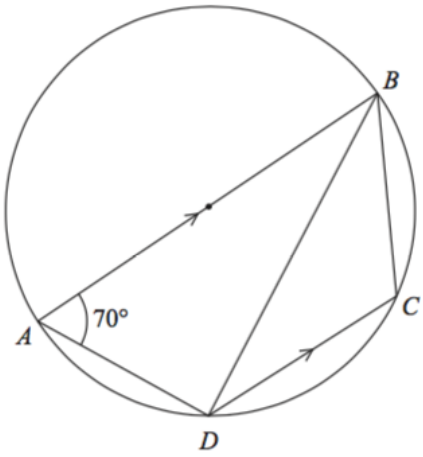
Categorisation: Construct extra lines based on given instructions.

[Edexcel IGCSE Jan2015(R)-4H Q16b]

A, *B*, *C* and *D* are points on a circle. *AB* is a diameter of the circle.
DC is parallel to *AB.* Angle *BAD* = 70°
The tangent to the circle at *D* meets the line *BC* extended at *T*.

Calculate the size of angle *BTD.*

Diagram **NOT** accurately drawn



..... °

Question 22

Categorisation: Determine the area of a segment, making use of circle theorems.

[Edexcel IGCSE Jan2012-3H Q18]

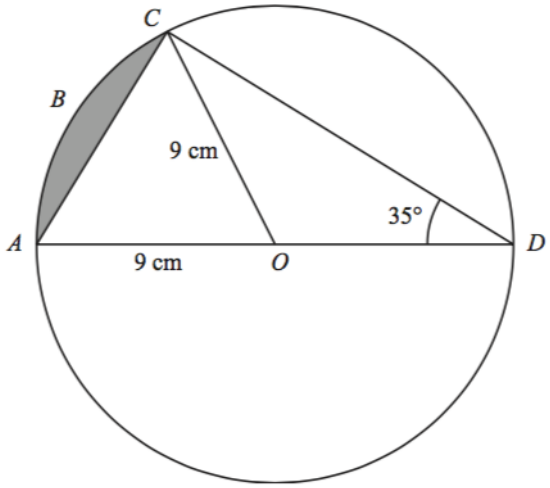


Diagram **NOT** accurately drawn

AOD is a diameter of a circle, with centre O and radius 9 cm.
ABC is an arc of the circle.
AC is a chord. Angle ADC = 35°

Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.

..... cm^2

Question 23

Categorisation: Prove the congruency of two triangles using circle theorems.

[Edexcel GCSE Nov2013-2H Q28 Edited]

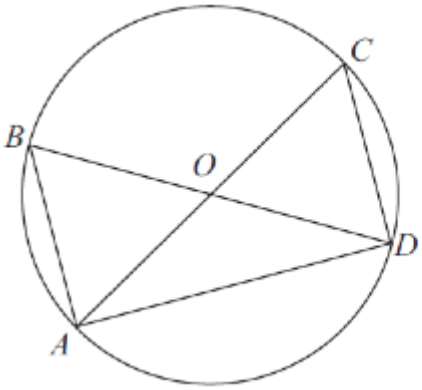


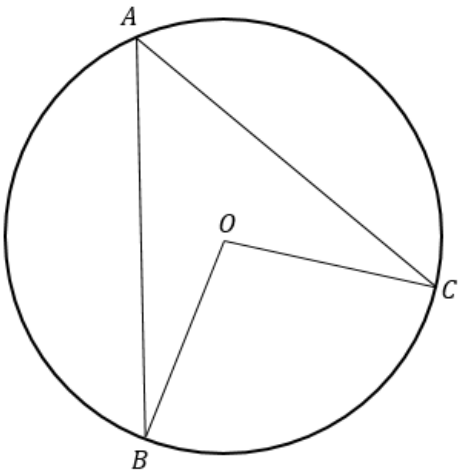
Diagram **NOT** accurately drawn

AOC and *BOD* are diameters of a circle, centre *O*.
Prove that triangle *ABD* and triangle *DCA* are congruent.

Question 24

Categorisation: Prove the circle theorems.

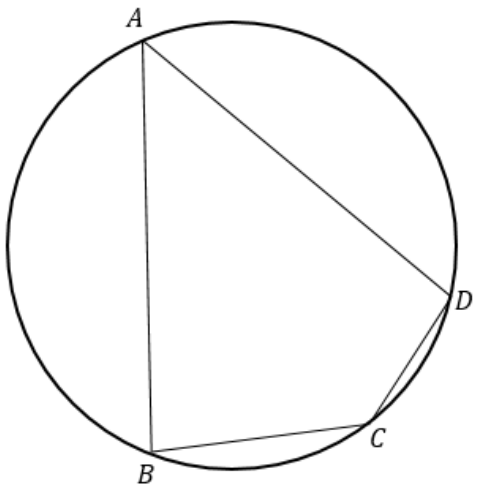
Prove that the angle at the centre of a circle is twice that angle at the circumference. You may not refer to other circle theorems.



Question 25

Categorisation: Prove the circle theorems.

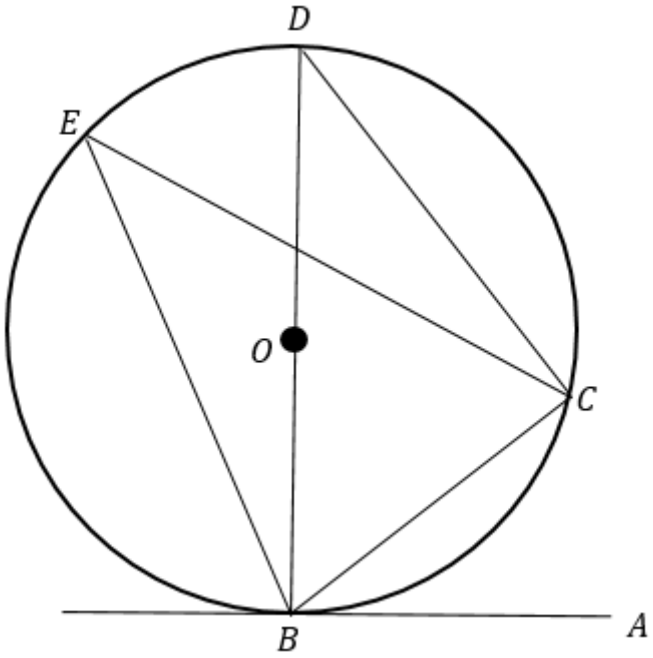
Prove that opposite angles of a cyclic quadrilateral add to $180°$.



Question 26

Categorisation: Prove the circle theorems.

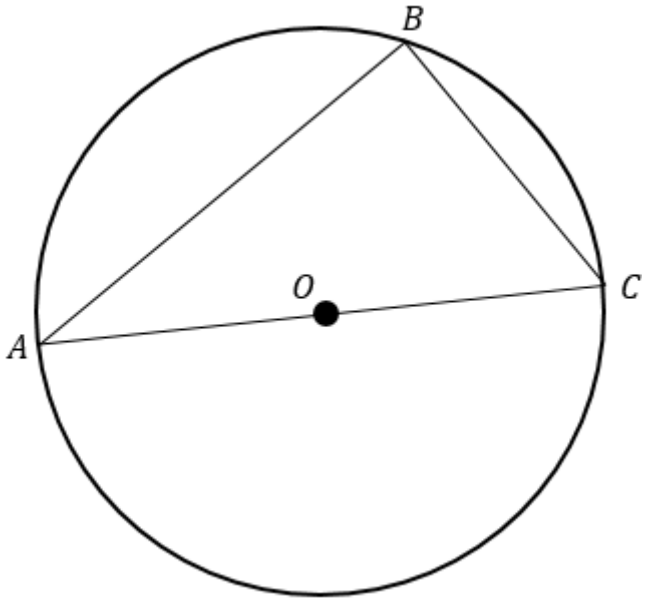
Prove the Alternate Segment Theorem. You may find the diagram below helpful, proving that angle CBA = angle CEB.



Question 27

Categorisation: Prove the circle theorems.

Prove that angles in a semicircle are equal to $90°$.



Answers

Question 1

116 °

Question 2

50 °

Question 3

Opposite angles of cyclic quadrilateral add to $180°$.
Angle at centre is twice angle at circumference.

Question 4

74 °

Question 5

angle $BCD =$ 65 °

Question 6

Angle ODC = 65 °

Question 7

75 °

Question 8

34 °

Question 9

27°

Question 10

Tangents from a point to a circle are equal in length

Question 11

"63" and “Alternate Segment Theorem” (other alternatives possible)

Question 12

23 °

Question 13

angle $RQS =$ 54 °

Question 14

70 °

Question 15

48 °

Question 16

83 °

Question 17

$\angle STA =$ 47 °

Question 18

35 °

Question 19

$\angle AOB =$ 66 °

Question 20

Angle $ABC = 180 - \frac{y}{2}$

Question 21

60 °

Question 22

11.4 cm^2

Question 23

Any of SAS, ASA and RHS possible, using an appropriate combination of three of the following:

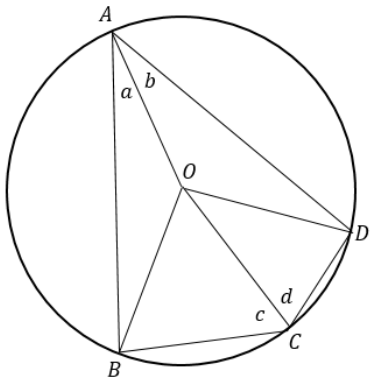
"AD is common",
"Angle BAD = angle CDA (angles in a semicircle are 90°)"
"Angle ABD = angle DCA (angles in the same segment are equal)"
“Angle OAD = angle ODA (base angles of isosceles triangle are equal”
“AC = BD (both are diameters)”

The proof should finish with a statement of which congruency proof was used, e.g. “Therefore ABD and DCA are congruent by SAS”.

Question 24

(Draw a line from A to O)
Let $\angle BAO = x$ and $\angle OAC = y$.
$\angle ABO = x$ and $\angle ACO = y$ (base angles of an isosceles triangle are equal)
$\angle AOB = 180 - 2x$ and
$\angle AOC = 180 - 2y$ (angles in a triangle sum to $180°$)
Thus $\angle BOC = 360 - (180 - 2x) - (180 - 2y) = 2x + 2y = 2(x + y) = 2 \times \angle BAC$

Question 25



Consider the diagram. $\angle OBC = c, \angle ODC = d, \angle ADO = b$ and $\angle ABO = a$ (base angles of isosceles triangles are equal).
Then considering angles in quadrilateral $ABCD$:

$$a + b + b + d + d + c + c + a = 360°$$
$$2a + 2b + 2c + 2d = 360°$$
$$(a + b) + (c + d) = 180°$$

Therefore $\angle BAD + \angle BCD = 180°$

Question 26

Let angle $CBA = x$. $\angle DBC = 90 - x$ (angle between radius and tangent is $90°$). $\angle DCB = 90°$ (angle in semicircle is $90°$). Therefore $\angle BDC = 180 - 90 - (90 - x) = x$.
$\angle BEC = x$ (angles in same segment are equal). Thus $\angle CBA = \angle BEC$.

Question 27

Let $\angle BCO = x$. Then $\angle OBC = x$ (base angles of isosceles triangle are equal).
$\angle BOC = 180 - 2x$ (angles in triangle sum to $180°$) therefore $\angle AOB = 2x$ (angles on straight line add to $180°$). $\angle BAO = \angle ABO = \frac{180-2x}{2} = 90 - x$ (base angles of isosceles triangle are equal).
$\angle ABC = \angle ABO + \angle OBC = (90 - x) + x = 90°$ therefore angle in semicircle is $90°$.