GCSE – Laws of Indices

Exercise 1a

1. Simplify the following.
   a. $x^2 \times x^3$
   b. $(x^2)^3$
   c. $\frac{m^{20}}{m^4}$
   d. $y^8 \times y^{-2}$
   e. $q \times q^3$
   f. $(p^5)^2$
   g. $x \times x^3 \times x^5$
   h. $\frac{y^6}{y^2}$
   i. $x^2 \times x^a$
   j. $(x^2)^y$
   k. $\frac{x^{12}}{x^3}$
   l. $\frac{x^{12}}{x}$
   m. $(p^{-q})^{-r}$
   n. $\frac{w^4}{w^{-4}}$
   o. $\frac{a^b}{a^{-b}}$
2. Simplify the following.
   a. $y \times \frac{y^{10}}{y^5}$
   b. $(x^2 \times x^3)^2$
   c. $\left(\frac{p^{12}}{p^3}\right)^4 \times p$
   d. $\frac{x^{10} \times x^9}{(x^5)^3}$
   e. $(q^2)^3 \times \left(\frac{q^5}{q}\right)^2$
   f. $\left(\frac{x^{2y}}{x^y}\right)^3$
   g. $x \times (x \times (x^3)^2)^2$
3. If $\frac{2^x \times 2^y}{2^3} = 2^7$, what is x in terms of y?
4. Simplify the following:
   a. $2x^2y \times 3xy^4$
   b. $5p^3q^4 \times 5pq$
   c. $\frac{10x^{10}y}{2xy}$
   d. $\frac{36k^3m^4}{30k^5m}$
   e. $\frac{2xy \times 2x^{10}}{8x^{20}}$
5. *[Edexcel GCSE June2003-6H Q17a]*
   If $x = 2^p$, $y = 2^q$, express the following in terms of x and/or y:
   (i) 2^{p+q}
   (ii) 2^{2q}
   (iii) 2^{p-1}
6. Simplify the following.
   a. $3^y + 3^y + 3^y$
   b. $2^{2y} + 2^{2y}$
   c. $(2^x + 2^x)^2$

☠ Solve $\frac{(2^x)^5}{2^3} = \frac{2}{(2^4)^x}$

☠ Given that $x^{\frac{1}{2}} = \sqrt{x}$, express $4^x + 4^x$ as a single power of 4.

Exercise 1b

1. Determine the value of:
   a. 6^{-1}
   b. 9^0
   c. $\mathbf{8^{-2}}$
   d. $\left(\frac{8}{9}\right)^{-2} = \mathbf{\frac{81}{64}}$
2. If $2^y = \frac{1}{4}$, write down the value of y.

☠ *[Edexcel GCSE June2003-6H Q17b]*
Let $x = 2^p$, $y = 2^q$,
If xy = 32 and $2xy^2 = 32$, find the value of p and the value of q .

Exercise 2

1. $100^{\frac{1}{2}}$
2. $125^{\frac{1}{3}}$
3. $16^{-0.5}$
4. $27^{-\frac{2}{3}}$
5. $8^{\frac{4}{3}}$
6. $8^{-\frac{1}{3}}$
7. $64^{-\frac{1}{3}}$
8. $64^{-\frac{2}{3}}$
9. $32^{\frac{2}{5}}$
10. $32^{-\frac{2}{5}}$
11. $\left(\frac{64}{27}\right)^{-\frac{1}{3}}$
12. $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$
13. $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$
14. $\left(\frac{8}{27}\right)^{-\frac{5}{3}}$
15. Write the following expression without using indices: $x^{-0.5}$

Exercise 3

1. $(xy)^2$
2. $(3x)^2$
3. $(xy^2)^2$
4. $(2cd^4)^3$
5. $(ab^2)^3$
6. $(9a^2)^{\frac{1}{2}}$
7. $(16a^4b^3)^{\frac{1}{2}}$
8. $(27a^9b^4)^{\frac{1}{3}}$
9. $(8a^6b^{12})^{\frac{2}{3}}$
10. $(16a^6b^{12})^{\frac{3}{2}}$
11. $(27x^6y^5)$

Exercise 4 (intended for FM students)

1. If $x^{\frac{2}{3}} = 9$, find x.
2. Solve $y^{\frac{4}{3}} = 16$
3. [AQA FM June 2012 Paper 1] $x^{\frac{3}{2}} = 8$ and $y^{-2} = \frac{25}{4}$.
   Work out the value of $\frac{x}{y}$
4. [AQA FM June 2013 Paper 1]
   Solve $x^{-\frac{2}{3}} = 7\frac{1}{9}$ writing your answer as a proper fraction.
5. [June 2013 Paper 2] $p^{-2} = q^6 \times r^4$
   Write p in terms of q and r.
   Give your answer in its simplest form.
6. [AQA FM Set 3 Paper 1]
   $x^{\frac{1}{2}} = 6$ and $y^{-3} = 64$
   Work out the value of $\frac{x}{y}$
7. [AQA FM Set 1 Paper 2] You are given that $x = 5^m$ and $y = 5^n$.
   a. Write 5^{m+2} in terms of x.
   b. Write 5^{m-n} in terms of x and y.
   c. Write 5^{3n} in terms of y.
   d. Write $5^{\frac{m+n}{2}}$ in terms of x and y.

Exercise 5 (intended for all Higher Tier students)

1. Write as a single power of 2:
   a. 4^{10}
   b. $8^x \times 2^4$
   c. $16^{\frac{2}{3}}$
   d. $2^x \times 4^y \times 8^z \times 16$
   e. $2\sqrt{2}$
   f. $\frac{4}{\sqrt[3]{2}}$
   g. $4^{\frac{1}{3}} \times 8^{\frac{1}{5}}$
2. *[Edexcel GCSE(9-1) Nov 2017 2F Q21c, Nov 2017 2H Q6c Edited]* $100^a \times 1000^b$ can be written in the form 10^w
   Express w in terms of a and b.
3. Solve for x:
   a. $8^x = 2^9$
   b. $5^x = 5\sqrt{5}$
   c. $4^x = \frac{8^6}{2^4}$
   d. $27^x = \sqrt{9^{30}}$
   e. $4^{2x+1} = 8^{2x-1}$
4. *[Edexcel GCSE(9-1) June 2017 2H Q18]*
   $$16^{\frac{1}{5}} \times 2^x = 8^{\frac{3}{4}}$$
   Work out the exact value of x.
5. *[Edexcel IGCSE Jan2017-4H Q16d]*
   $$\frac{1}{\sqrt[3]{9^4}} = 3^n$$
   Work out the exact value of n.

☠ Solve
$\sqrt[3]{9} \times \sqrt[4]{27} = \sqrt[x]{3}$