30-4-10 Shape and Space
DRAFT

Topic: 3D Shapes and Nets

This is not a grade C topic as such but an understanding of 3D shapes and nets is needed in order to

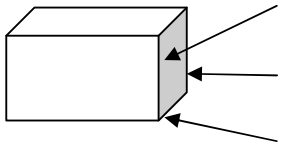
- Solve problems involving surface areas and volumes of right prisms.

This unit of work can be used to recap 3D shapes and nets prior to further work on surface area and volume.
You need to be able to:

- Visualise and use 2D representations of 3D objects.
- Analyse 3D shapes through 2D projections including plans and elevations.
- Construct specified cubes, regular tetrahedrons, square based pyramids and other 3D shapes.

You will need to think about:

- Faces, vertices and edges

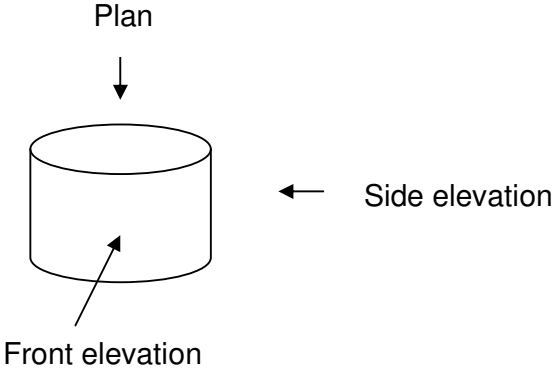


A **face** is a flat surface of a solid

An **edge** is the line where two faces meet

A **vertex** is a point where three or more edges meet

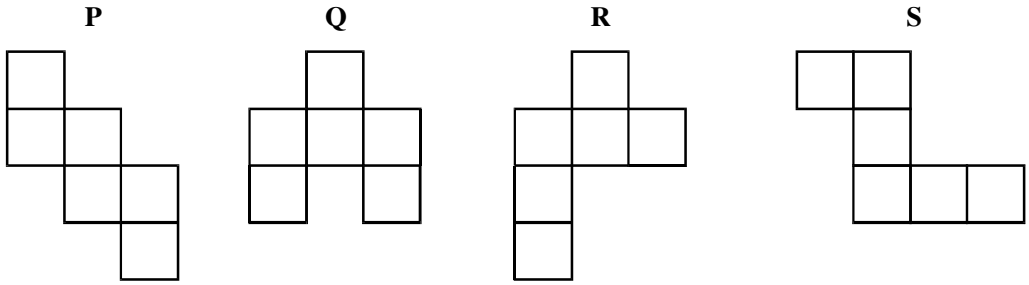
- 3D shapes: cube, cuboid, prism, pyramid, tetrahedron, sphere, cylinder, cone
- A net is a 2D shape that can be folded to form a 3D shape
- A solid can be viewed from different directions:
  - A **plan** view (from above), a **front elevation** and a **side elevation**



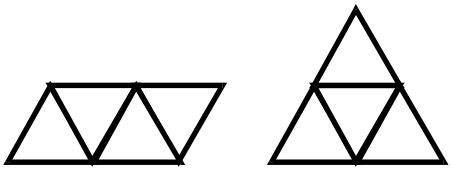
Quick Questions

1. Sketch each of these solids and write down the number of faces, vertices and edges for each one.
   a) Triangular prism
   b) Square based pyramid
   c) Cuboid
   d) Hexagonal prism

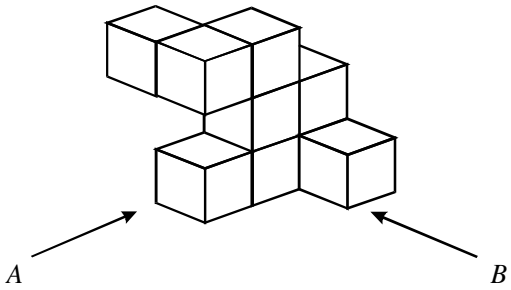
2. Which of these is the net of a cube?



3. Name the solid that can be made from these nets

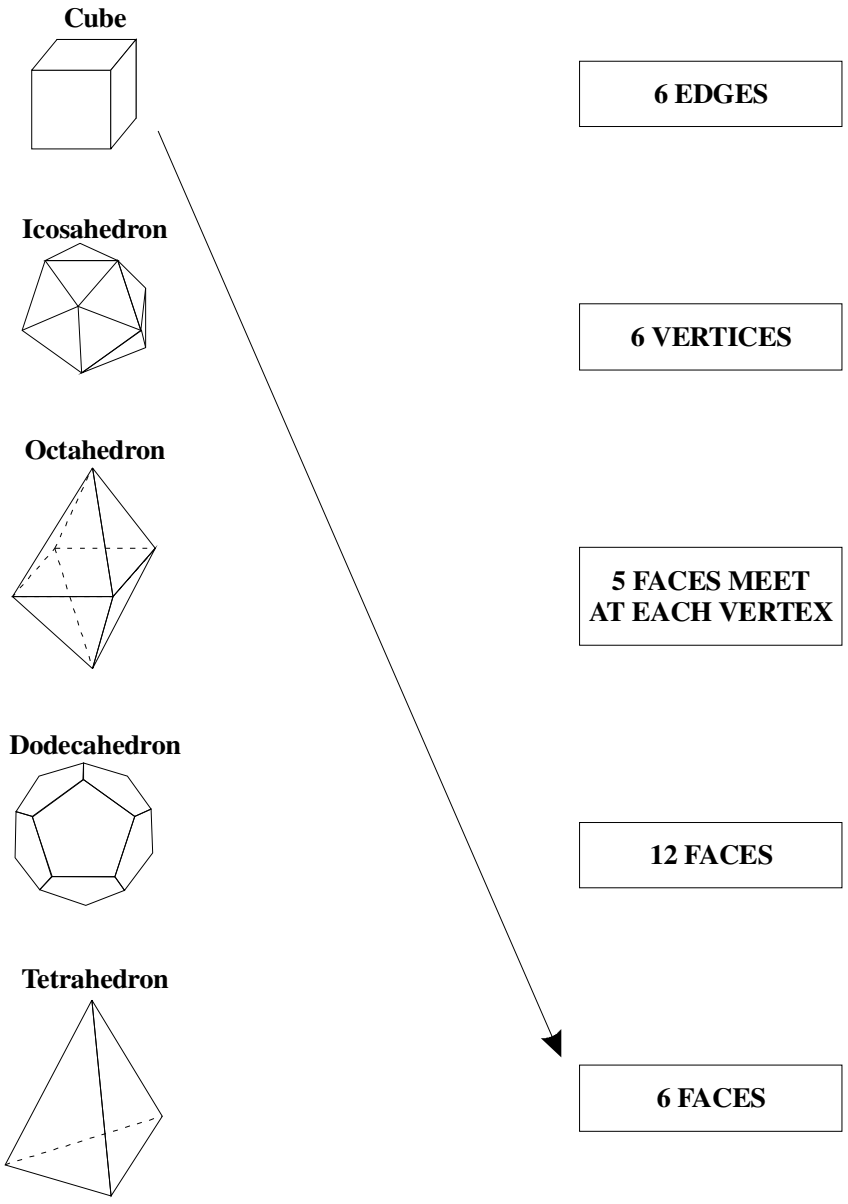


4. The diagram shows a solid made from 9 small cubes. Draw
   a) a plan view of the solid
   b) a view of the solid from direction A
   c) a view of the solid from direction B



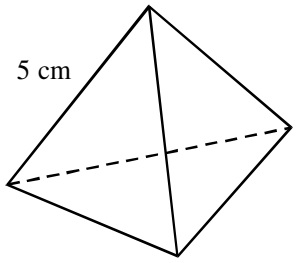
Past Examination Questions *(From AQA GCSE papers)*

1. (a) Five regular solid shapes are shown below.
Also given are five descriptions of the solids.
Match each description to the correct solid by drawing an arrow.
The cube has been done for you.



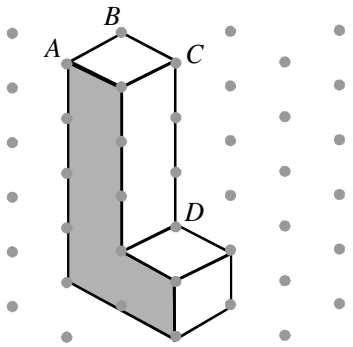
(2)

2. The diagram shows a regular tetrahedron.
Each edge is 5 cm long.
Draw an accurate net of the tetrahedron.



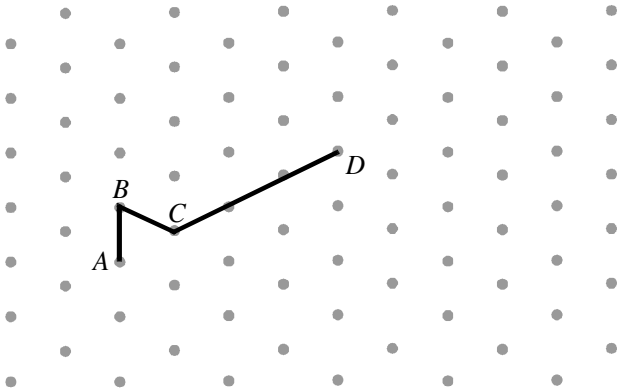
(3)

3. The solid shape shown falls over onto the shaded face.



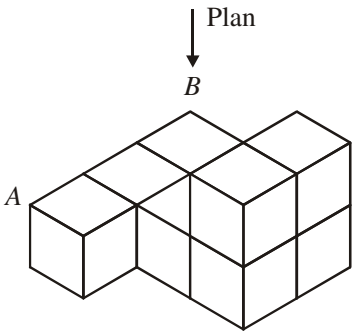
On the grid below, draw the shape after it has fallen over.

The lines *AB, BC* and *CD* have been drawn for you.

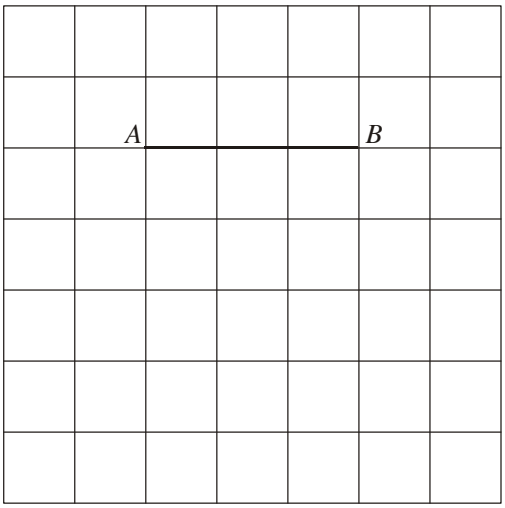


(3)

4. The diagram shows a solid shape made from 8 cubes.

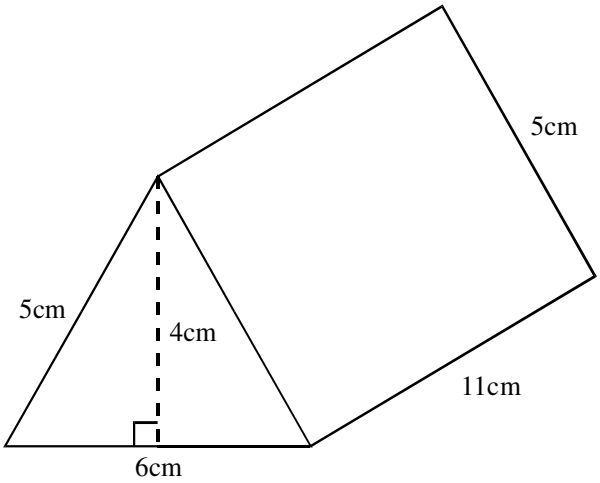


Complete the plan view of the shape on the grid below.



(Total 2 marks)

5. A triangular prism has dimensions as shown.



(a) **Sketch** a net of the prism.
*(You do **not** need to draw an accurate diagram.)*

(1)

(b) Calculate the total surface area of the prism.
Show all your working.

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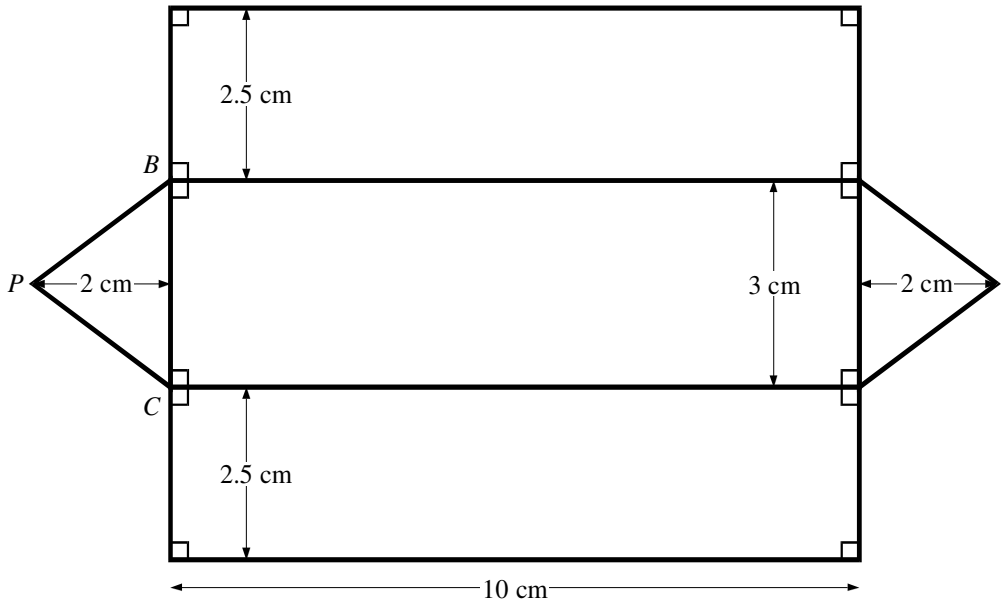
.....

.....

(4)

6. This net will fold to make a three-dimensional shape.



(a) Measure the size of the angle *BPC*.

.....

(1)

(b) Calculate the area of the net.
(Remember to state the units in your answer.)

.....

.....

(3)

(c) (i) Draw a sketch of the shape that the net makes when folded.

(1)

(ii) What is the mathematical name of this shape?

.....

(1)

(ii) Calculate the volume of this shape.
(Remember to state the units in your answer.)

.....

.....

(3)

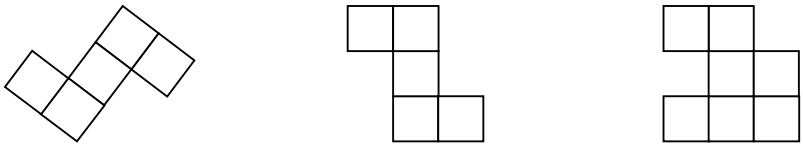
Answers

Quick Questions

1. Sketches

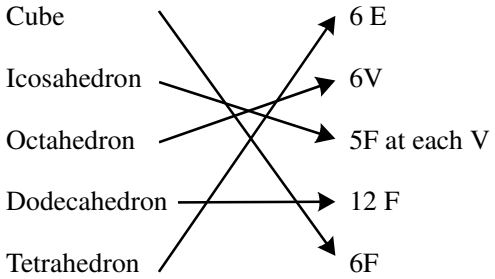
	Faces	Vertices	Edges
Triangular prism	5	6	9
Square based pyramid	5	5	8
Cuboid	6	8	12
Hexagonal prism	8	12	18

2. P
3. Tetrahedron
4. a) b) c)



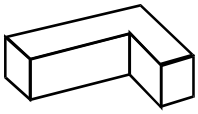
Examination Questions

1.

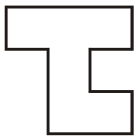


2. Correct net

3.



4.



5. a) A correct net showing three rectangles and two triangles

b) Area triangle = ½ × 6×4 (×2) or 12 (×2) or 24

Rectangle = 11×5 (×2) or 55 (×2) or 110

Rectangle = 11 × 6 or 66

S.A. = 200cm²

6. (a) 71 – 75°

(b) 10 X 8 + ½ X 3 X 2 X 2

80 + 6 = 86cm^2

(c) (i) sketch of a triangular prism

(ii) triangular prism

(iii) Area of cross-section X length = ½ X 3 X 2 X 10 = 30 cm^3

30-4-10 Shape and Space
DRAFT

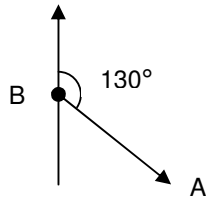
Topic: Bearings

You need to be able to:

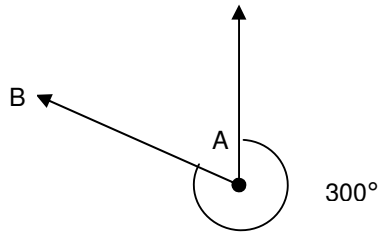
- Understand and be able to solve problems involving bearings
- use angle measure [for example, use bearings to specify direction]

You will need to think about:

Bearings are used to describe the direction in which you must travel to get from one place to another. They are measured from the North line in a clockwise direction. A bearing can be any angle from 0° to 360° and is written as a 3-figure number.



A is on a bearing of 130° from B

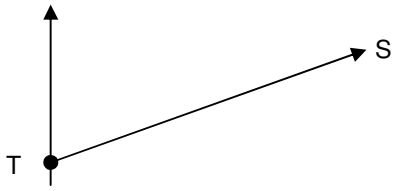


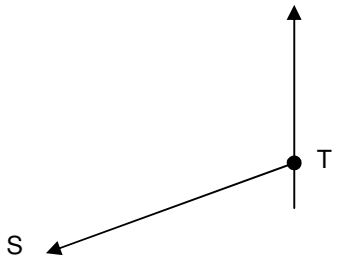
The bearing of B from A is 300°

Quick Questions

1. Draw diagrams to show the following:
    a) A is on a bearing of 145° from B.
    b) The bearing of P from Q is 230°

2. Find the bearing of S from T, in each of the following, by measuring.



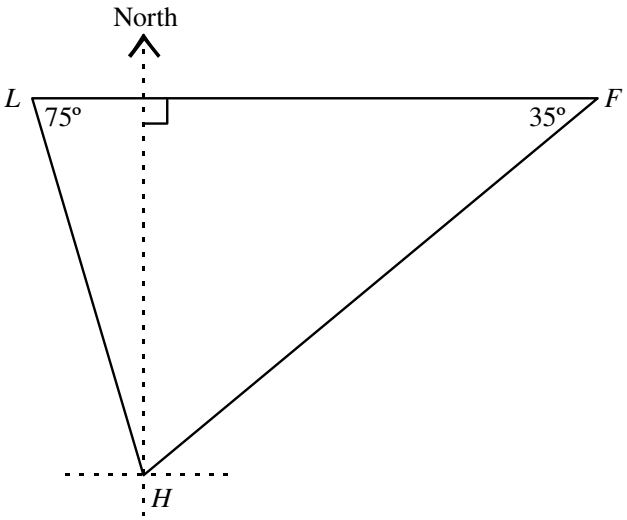


3. If the bearing of F from E is 036° what is the bearing of E from F?

4. An aeroplane flies from an airport A on a bearing of 022° until it reaches B, 75km away. It then turns so that it is travelling on a bearing of 222° towards C, 80km away. Using a scale of 1cm = 10km, make an accurate drawing of the aeroplane's route. By measuring, find the bearing and distance from C to A.

Past Paper Questions *(From AQA GCSE papers)*

1. A fishing boat sails from a harbour *H* to a point *F*.
F is due east of a lighthouse *L*
Angle *FLH* is 75° and angle *LFH* is 35°.



(a) Calculate the bearing of *F* from *H*.

.....

.....

(1)

(b) Calculate the bearing of *L* from *H*.

.....

.....

(2)

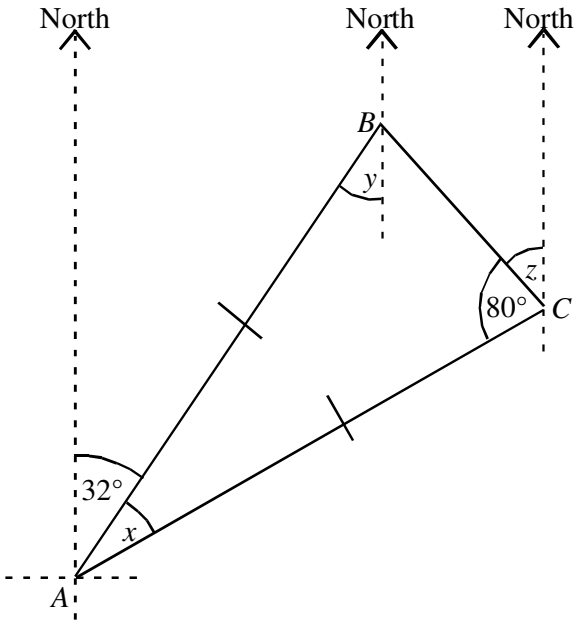
(c) Calculate the bearing of *H* from *F*.

.....

.....

(2)

2. The diagram shows the positions of three places *A*, *B* and *C*.



The diagram is not drawn to scale.

AB is the same length as *AC*.

(a) (i) Calculate the size of the angle marked *x*.

.....

.....

(1)

(ii) Explain why the angle marked *y* is equal to 32º.

.....

.....

(1)

(iii) Calculate the size of the angle marked *z*.

.....

.....

(1)

(b) Use your answers to (a) to calculate the bearing of

(i) *C* from *A*,

.....

(1)

(ii) *A* from *B*,

.....

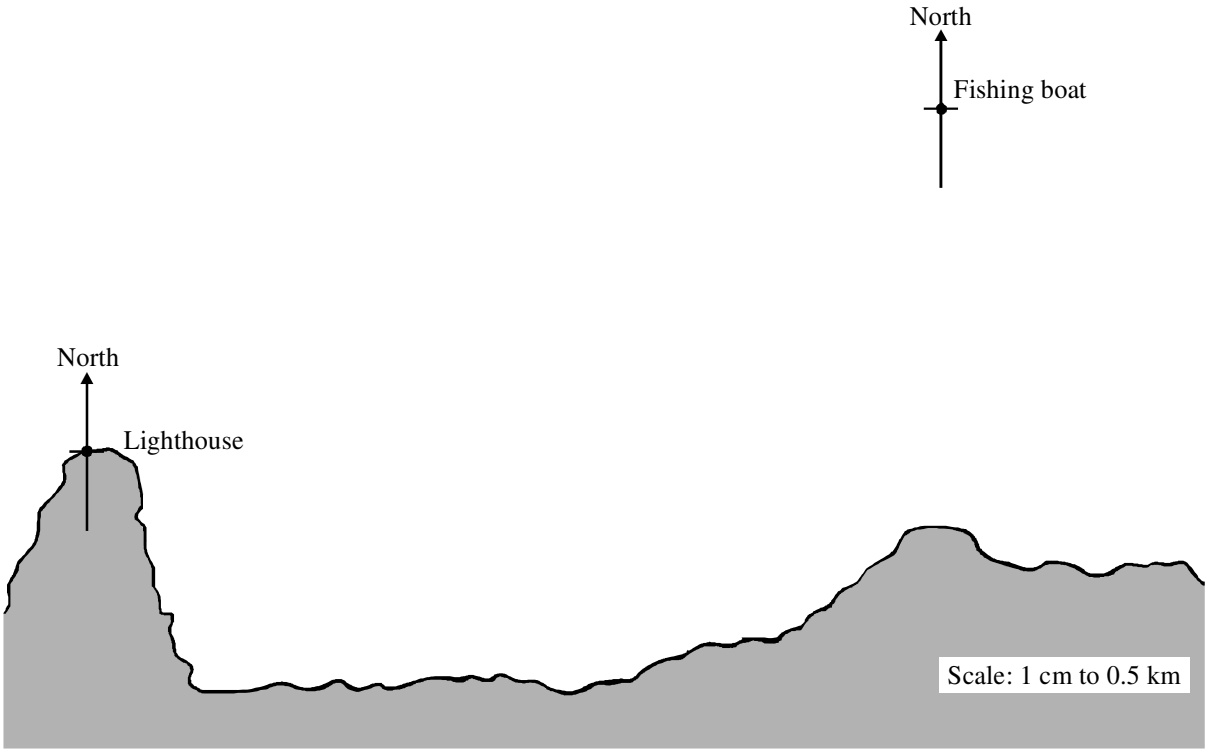
(1)

(iii) *B* from *C*.

.....

(1)

3. The map shows the position of a lighthouse and a fishing boat.
The map has been drawn to a scale of 1 cm to 0.5 km.



(a) What is the distance of the fishing boat from the lighthouse?
Give your answer in kilometres.

.....

.....

.....

Answer ..... km

(2)

(b) What is the bearing of the fishing boat from the lighthouse?

.....

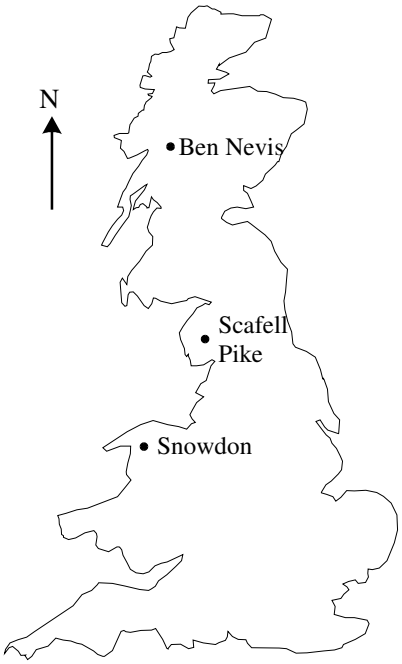
Answer ..... degrees

(1)

(c) A buoy is 5 km from the lighthouse and on a bearing of 200° from the fishing boat.
Mark, with an X, the position of the buoy on the map.

(2)

4. The map shows the location of the highest mountains in England, Scotland and Wales.



(a) Use your protractor to find the three figure bearing of Scafell Pike from Ben Nevis.

.....

.....

(2)

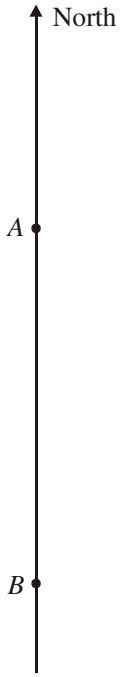
(b) Write down the three figure bearing of Ben Nevis from Scafell Pike.

.....

.....

(2)

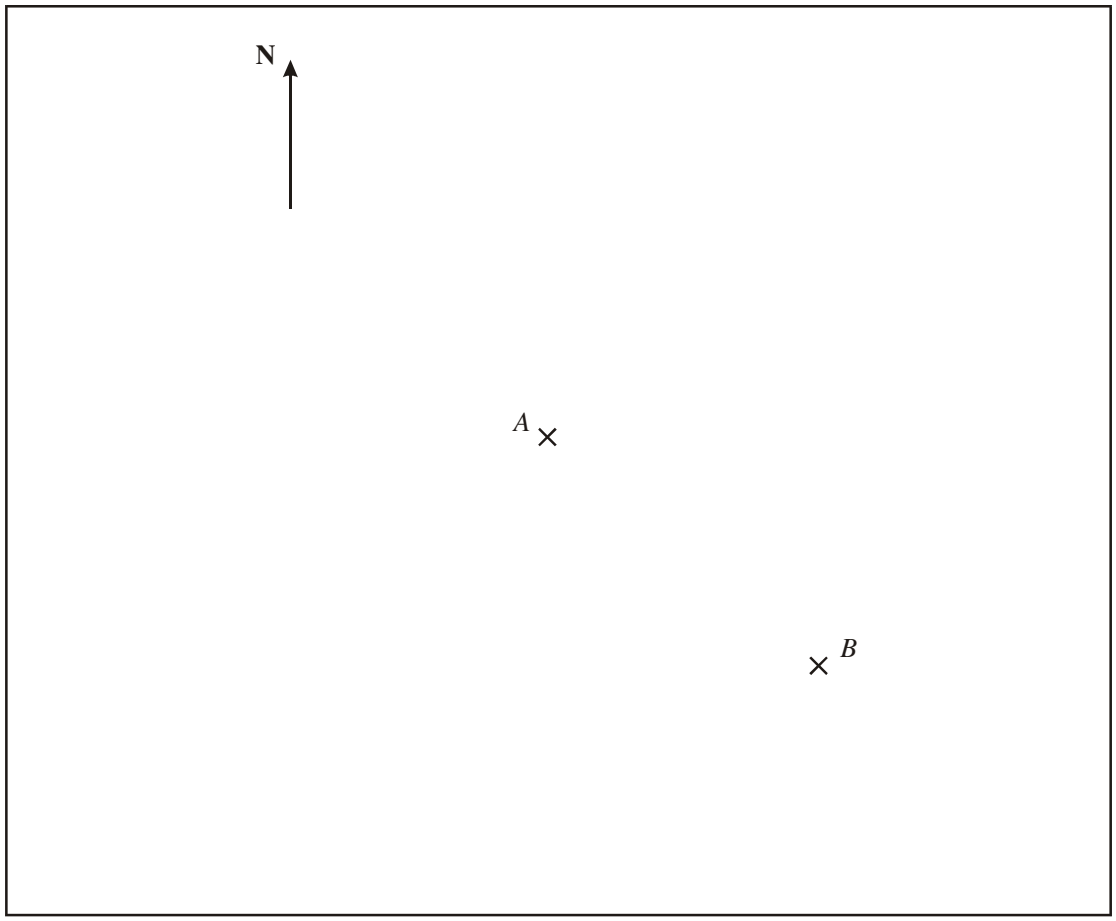
5. *A* is due North of *B*.
The bearing of *C* from *A* is 115°.
The bearing of *C* from *B* is 075°.



Mark the position of *C* on the diagram.

(Total 3 marks)

6. The diagram shows two points *A* and *B*.



(a) Measure the bearing of *B* from *A*.

.....

Answer ..... °

(1)

(b) Point *C* is South-West of *A* and West of *B*.
Mark the position of *C* on the diagram.

(2)

(Total 3 marks)

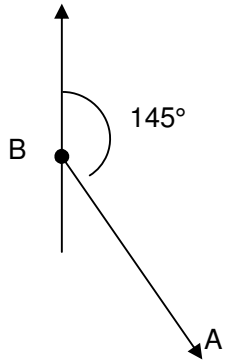
END OF QUESTIONS

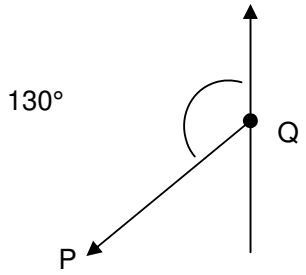
30-4-10 SSM **Answers**

Day: **Topic:**

Quick Questions

1





2. a, 070° b, 250°

3. 36 + 180 = 216°

4.

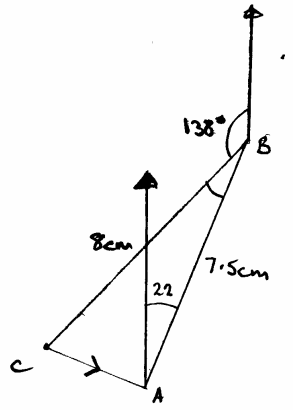


Diagram approx half scale.

Distance from C to A ~ 27.5 km

Bearing of C from A ~ 112°

Past Paper Questions

1. (a) 055° or N 55° *Accept 55°.* B1

(b) 360 – (180 – 90 – 75) or 180 – 90 – 75 then N x° W M1
345° N 15° W A1

(c) 180 + (90 – 35) or 90 – 35 then S x° W M1
235° S 55° W A1

*The method marks in **(b)** and **(c)** are for knowing which angle gives the required bearing.*

***Special Case:** If the angles are measured instead of calculated.*

(a) 50° ± 1° *(b) 343° ± 1°* *(c) 230° ± 1°*

B0 *B1* *B1*

[5]

2. (a) (i) $x = 20^\circ$ B1

(ii) Because of alternate angles. *Accept Z angles* B1
or $90 - [180 - 90 - 32] = 32^\circ$ *Do not accept corresponding angles*

(iii) $z = 80 - 32 = 48^\circ$ or $z = 180 - 80 - 32 - 20 = 48^\circ$ B1
Follow through using candidate's value of x.

(b) (i) $32 + 20 = 52^\circ$ *Accept 52° and also N 52° E.* B1
Bearing is 052° *Follow through using candidate's value for x.*

(ii) $180 + 32 = 212^\circ$ *Accept S 32° W.* B1
Bearing is 212°

(iii) $360 - 48 = 312^\circ$ *Accept N 48° W* B1
Bearing is 312° *Follow through using candidate's value for z.*

Special Case:
If a candidate measures the angles on the diagram, no marks can be given for (a).
All three marks may be earned for (b).
(b)(i) $60^\circ \pm 1^\circ$ (ii) $215^\circ \pm 1^\circ$ (iii) $319^\circ \pm 1^\circ$

Systematic Error:
If a candidate does all three bearings 'the wrong way round',
(i) A from C the bearing is $280 - z = 232^\circ$ B1
(ii) B from A the bearing is 032° (Accept 32°) B1
(iii) C from B the bearing is $180 - z = 132^\circ$ B1
Then make a 'systematic error' penalty of 2 marks.

[6]

3. (a) 13.6×0.5 M1
or 13.4 to 13.8
or (their 13.6) × 0.5

6.7 to 6.9 A1

(b) 062 to 066 B1
Allow 62 to 66

(c) Cross marked within limits of loci B2
or arc of circle
radius 10cm ±2mm
centre L B1
or X on bearing of 200 ±2 °from F B1

[5]

4. (a) 156 ..... 164 B2
150 – 155 & 165 – 170 B1
S (10 – 30) E, SSE B1

(b) $180 + \uparrow = 340$ M1 A1√

(344 – 336)

N (16 – 24) W etc B1√

√could be ± 180 + (a)

[4]

5. *C* marked within limits of loci B3

B1 bearing from A ± 2°

B1 bearing from B ± 2°

[3]

6. (a) 130 B1

±2°

(b) C in correct position ± 2mm B2

B1 if C south west of A ± 1°

B1 if C west of B ± 1°

[3]

END OF ANSWERS

30-4-10 Shape and Space
DRAFT

Topic: Constructions and loci

You need to be able to:

- draw approximate constructions of triangles and other 2-D shapes using a ruler and protractor, given information about their side lengths and angles; construct specified cubes, regular tetrahedra, square-based pyramids and other 3-D shapes
- use a straight edge and compasses to do standard constructions including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle
- find loci, both by reasoning and by using ICT to produce shapes and paths [for example, a region bounded by a circle and an intersecting line]

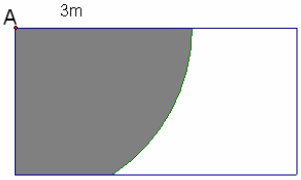
You will need to think about:

- a locus being a set of points (or line or region) that follows a given rule. The word loci is used when we talk about more than one locus.
- the way you use your compasses to find a point that is the same distance from two other places.

Examples of Loci:

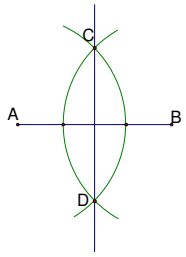
- The set of points equidistant from a given point in a plane is a circle.

- The shaded area is the locus of points within the rectangle that are less than 3m from corner A.



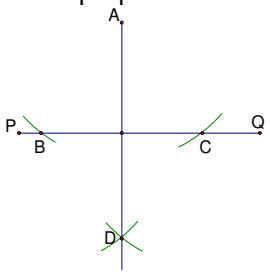
Using a ruler and compasses you should be able to

Construct the perpendicular bisector of a line



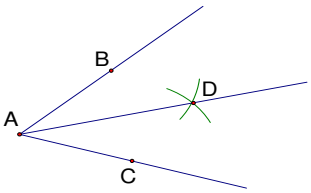
Points on line CD are equidistant from A and B

Construct the perpendicular from a point to a line



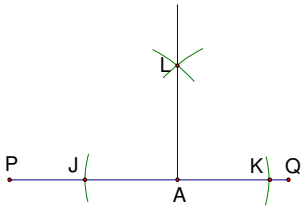
AD is the perpendicular from A to PQ

Construct the bisector of an angle



Points on the line AD are equidistant from the lines AB and AC

Construct the perpendicular from a point on a line

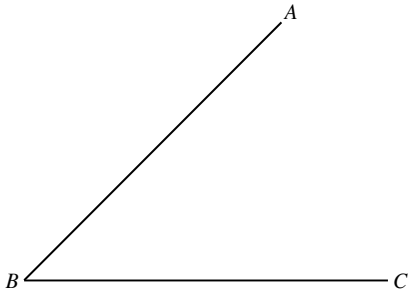


AL is perpendicular to PQ drawn from A

Quick Questions

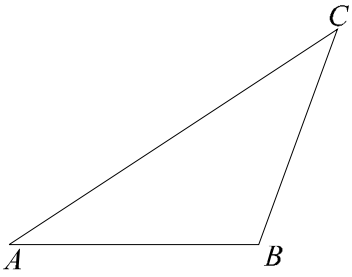
1. **Using ruler and compasses only**, construct an angle of 60°

2. **Using ruler and compasses only,** construct the bisector of angle *ABC.*



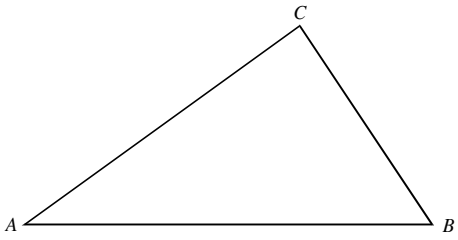
3. **Sketch** two possible nets for a regular tetrahedron.

   Choose one of the nets you have sketched. Draw this net accurately.

4. 

   The base *AB* of the triangle *ABC* is fixed.
   The point *C* can move, but the area of the triangle *ABC* stays the same.
   Describe, or draw, the locus of the point *C*.

5. The diagram shows a triangle, *ABC.*



   (a) Using a ruler and compasses only, construct the perpendicular bisector of *AB.*
   You **must** show clearly all your construction arcs.

   (b) Repeat this construction on another side of the triangle.

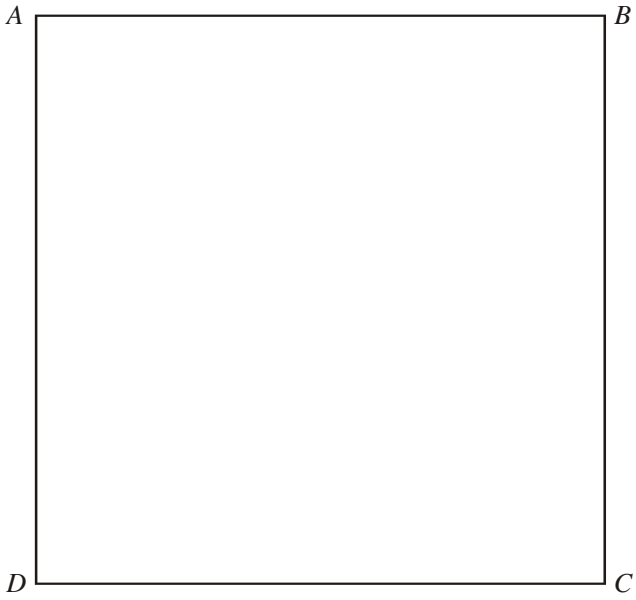
Past Paper Questions *(From AQA GCSE papers)*

1. In triangle *ABC*, the side *AB* is 7 cm.
Angle *A* = 40° and angle *B* = 95°.

Make an accurate drawing of the triangle in the space below.
The side *AB* has been drawn for you.

(Total 2 marks)

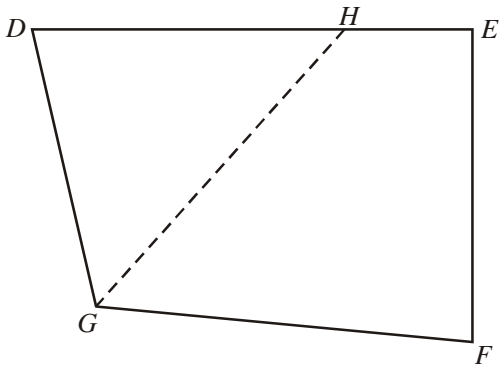
2. *ABCD* is a square of side 8 cm.



Show clearly the region inside the square that is both closer to the point *D* than to the point *A*, and closer to the side *CD* than the side *AD*.

(Total 3 marks)

3. The quadrilateral *DEFG* is a scale drawing of a field.
The line *GH* bisects angle *DGF*.



Scale:
1 cm represents 10 m

(a) Construct the locus of points in the field which are 40 m from *E*.

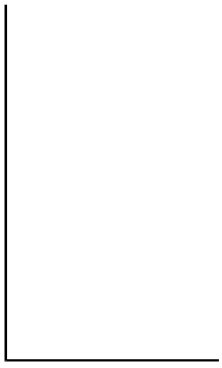
(1)

(b) Shade the area of the field which is more than 40 m from *E*
and nearer to *DG* than to *GF*.

(1)
(Total 2 marks)

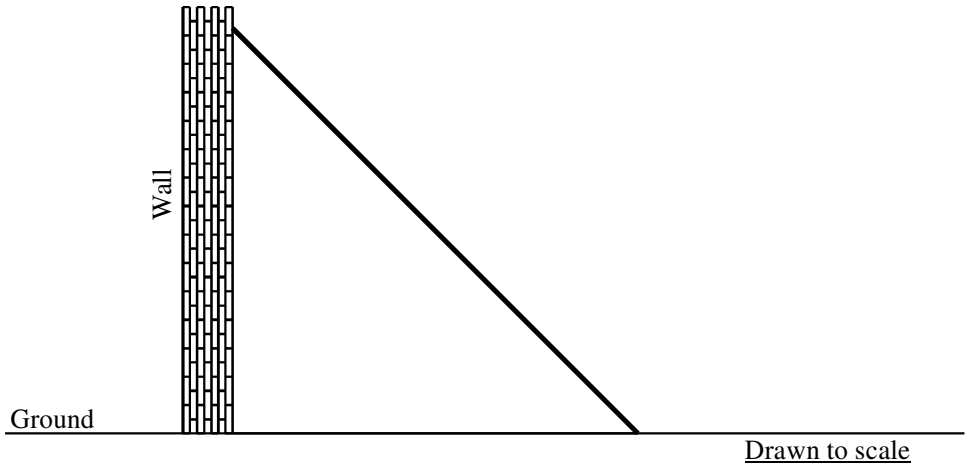
4. (a) A letter *L* is drawn as shown.
A point *P* is 2 cm from the letter *L*.

Draw the locus of all the possible positions of *P*.



(3)

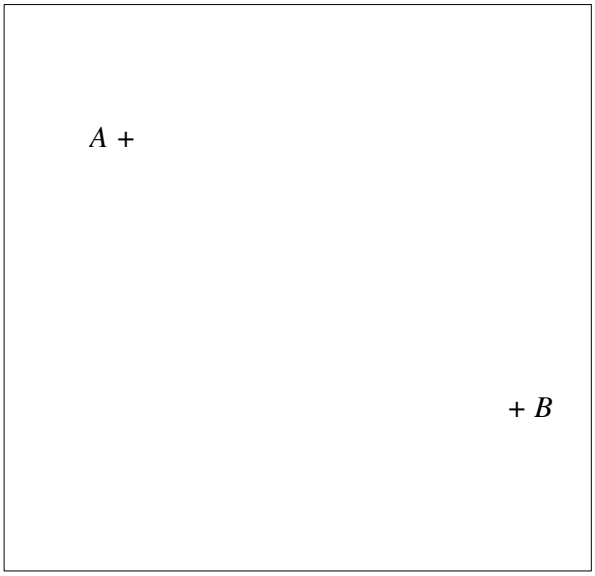
5. A ladder is 16 feet long.
Starting from the position shown, the ladder slips outward from the wall with its ends in contact with the wall and the ground.
Draw five possible positions of the ladder.
Hence draw the path of the centre of the ladder.



(3)

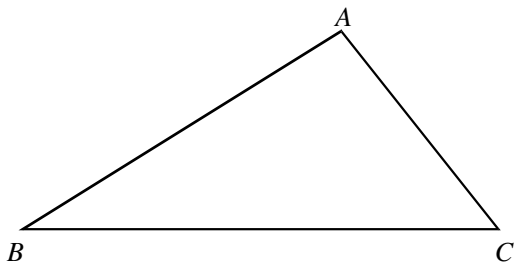
6. Two ships *A* and *B* both hear a distress signal from a fishing boat.
The positions of *A* and *B* are shown on the map below.
The map is drawn using a scale of 1 cm to represent 1 km.
The fishing boat is less than 4 km from ship *A* and is less than 4.5 km from ship *B*.
A helicopter pilot sees that the fishing boat is nearer to ship *A* than to ship *B*.
Use accurate construction to show the region which contains the fishing boat.
Shade this region.

Scale 1 cm = 1 km



(4)

7. ***To score full marks an accurate drawing is required.***
You may use any of your mathematical instruments to help you do this.



(a) On the diagram above

(i) draw the locus of a point which is the same distance from *A* as it is from *B*,

(2)

(ii) draw the locus of a point which is 6 cm from *C*.

(2)

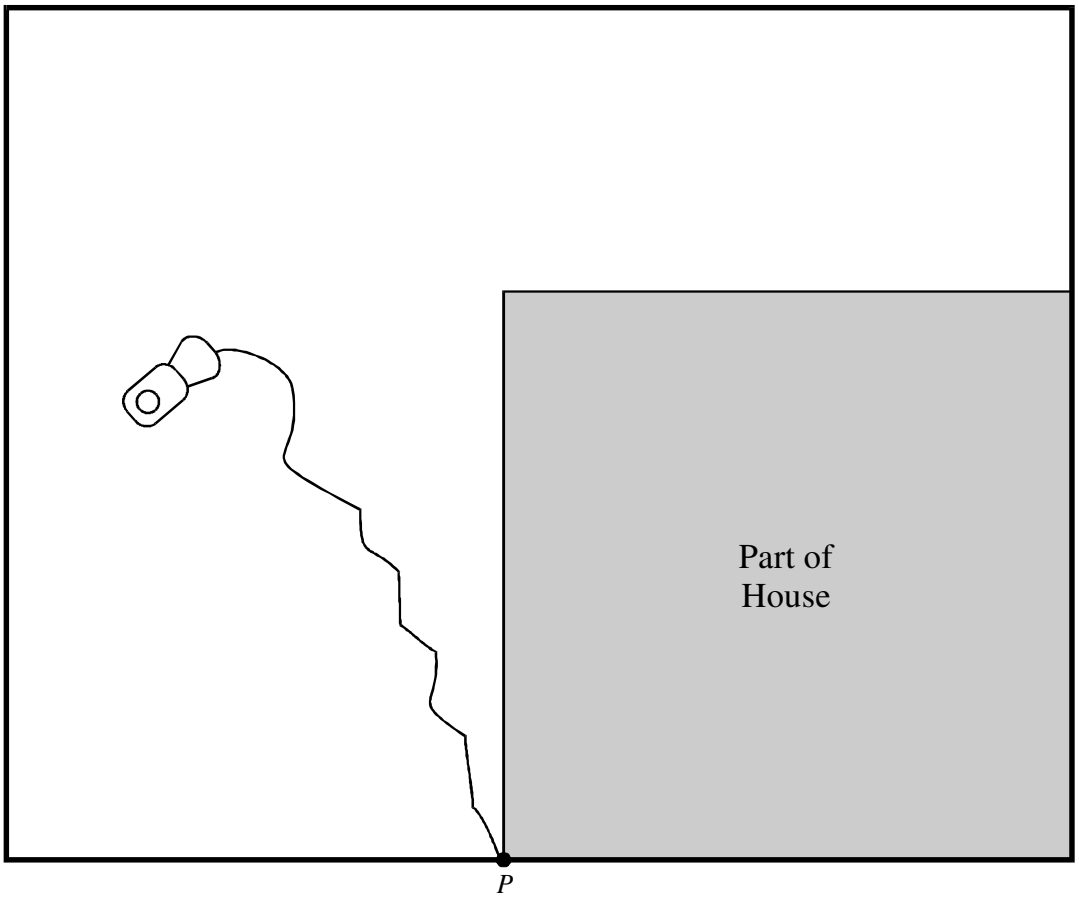
(b) *P* is a point inside the triangle *ABC*.
It is nearer to *B* than it is to *A*.
It is less than 6 cm from *C*.

Shade the region where *P* can be.

(1)

8. The scale diagram below shows a plan of Paul's garden.
Paul has an electric lawn mower.
The lawn mower is plugged in at point *P*. It can reach a maximum distance of 12 metres from *P*.

Scale: 1 cm represents 1 m

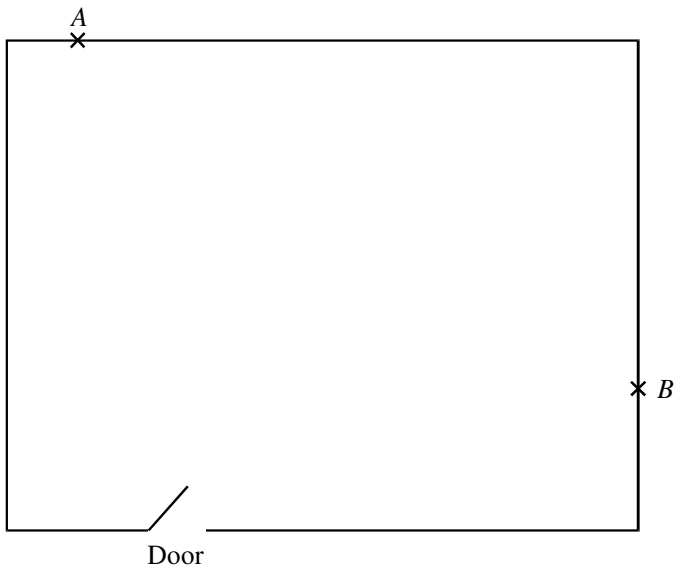


Using the same scale, show the area of the garden which the lawn mower can reach.

(3)

9. The scale diagram below shows a plan of a room.
The dimensions of the room are 9 m and 7 m.

Two plug sockets are fitted along the walls.
One is at the point marked *A*. The other is at the point marked *B*
A third plug socket is to be fitted along a wall.
It must be equidistant from A and *B*.
Using ruler and compasses, find the position of the new socket. Label it C.

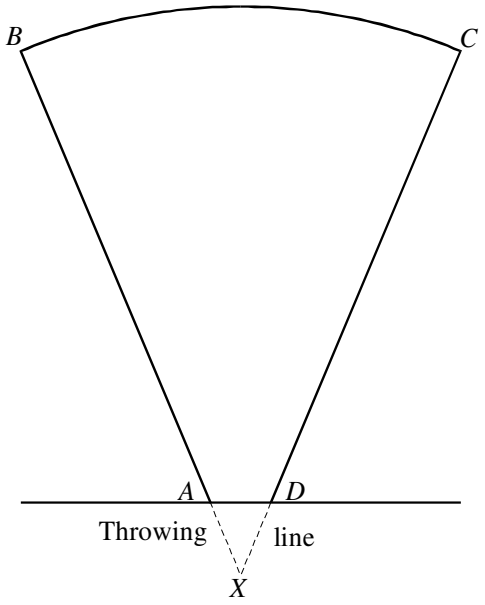


DRAWN TO SCALE

1 cm represents 1 m

(4)

10. The plan shows the landing area, *ABCD*, for a javelin event.
AD is the throwing line.
The arc *BC* is drawn from the centre *X*.
The plan has been drawn to a scale of 1 cm to 5 m.



Scale 1 cm to 5 m

The landing area is fenced off in front of the throwing line.
The position of the fence is always 10 m from the boundaries *AB*, *BC* and *CD* of the landing area.

Draw accurately the position of the fence on the plan.

(4)

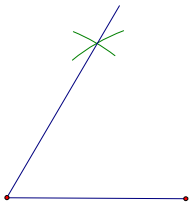
END OF QUESTIONS

30-4-10 SSM **Answers**

Day: **Topic:** Constructions

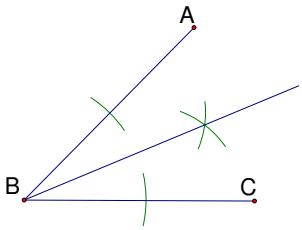
Quick Questions

1.

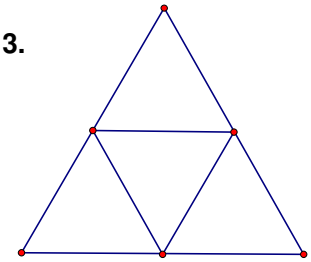


each arc drawn from end of line with radius = line length

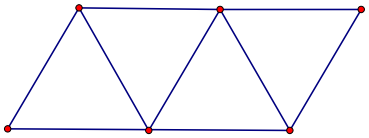
2.



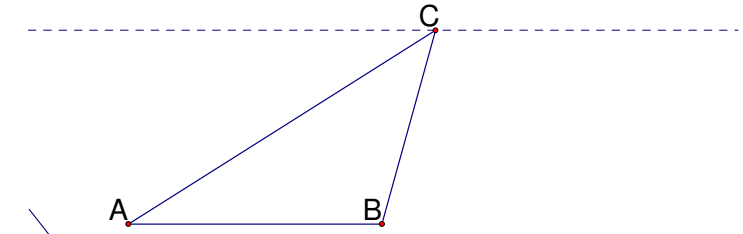
3.



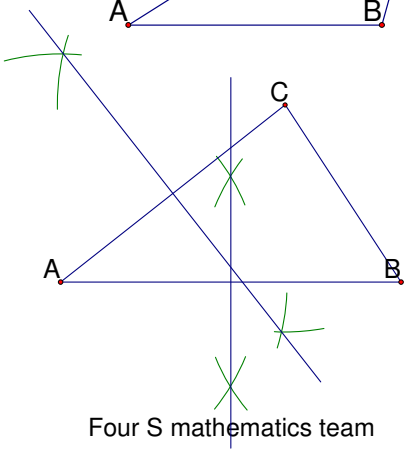
or



4. The locus of C is a straight line parallel to AB:



5.



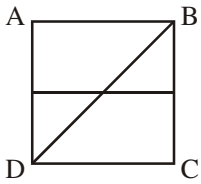
Past Paper Questions

1. Angle 95° or angle 40° ± 2° B1

Correct triangle B1

[2]

2.



B3

Line bisecting AD B1 at least 4 cm long
Line bisecting ADC B1 at least 4 cm long
Region shaded or marked B1 – trapezium, base DC, in bottom half of square.

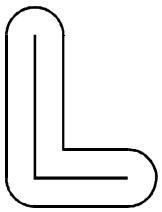
[3]

3. (a) Arc of radius 4 cm, centre E B1
4 cm ± 2 mm

(b) Correct area shaded – between GD and GH but outside arc. B1
(a) must show attempt at arc

[2]

4. (a)



B3

Allow loci within ±2mm
Freehand diagram within limits
ft incorrect scale
deduct B1 from mark
At least 12 dots/dashes covering all regions ±2mm B1 only
If additional lines are seen deduct B1 unless outside edge clearly identified as locus
or 5 correct parts (not freehand) B2
or 4 straight (not freehand) B1
or one semi-circle with compasses B1
or quadrant with compasses B1

[3]

5. 5 new positions below given ladder (including ground)
(8cm ± 2mm) B2
only 3 or 4 positions B1
Locus, arc of circle, centre junction of wall and ground, radius half length of ladder. B1
accept centres of ladder joined by straight lines but not an overall straight line locus.

[3]

6. Idea of a circle (arc) around A or B B1
both circles accurate with 2 intersections *(±1mm)* A1
Perpendicular bisector (±1mm centre ±2°) *accurately drawn or constructed* B1
Correct region shaded

must be from 3 lines as diag. B1√
2 arcs and a line through.

[4]

7. (a) (i) For knowing to draw the perpendicular bisector of AB. M1
For drawing the perpendicular bisector of AB accurately. A1
Through midpoint ± 1 mm 90° ± 1°
≥ 2 cm in length

(ii) For knowing to draw a circle centre C. M1
ie See arc cutting Δ, r = 6 cm ± 1 mm complete

For drawing the circle radius 6 cm accurately. A1

(b) For shading in the correct region. B1 cao
[5]

8. arc radius greater than current position of lawn mower M1
quarter circle radius M1
Correct, left arc ± 2mm
right arc ± 1mm A1
[3]

9. Arc draw from A or B, > 47mm radius M1
NB *no arcs, no M marks*

Equal arc drawn from B or A M1
2 cuts required unless midpoint of AB used

Perpendicular drawn or used M1 dep
2.3–- 2.5 cm from right hand corner

C identified correctly (full construction) A1
Award SC B1 if C correct with no valid arcs (i.e. M0)
[4]

10. Correct position within limits of loci
Compasses used for arcs B4
Any 4 correct parts B3
[Allow dots or dashes or freehand curves but not straight lines] max B3
or Straight line // AB or DC between 1.8 and 2.3 cm B1
Arc BC drawn BX + (1.8 to 2.3 cm) B1
Arcs at B or C radius (1.8 to 2.3 cm) B1
Incorrect scale –B1
[4]

END OF ANSWERS

Enabling world-class education

30-4-10 Shape and Space
DRAFT

Topic: Enlargement & Similarity

You need to be able to:

- Enlarge a shape from a given centre and with a given scale factor
- Recognise that CONGRUENT shapes are identical to each other
- Recognise that when shapes are SIMILAR one is an enlargement of the other
- Understand that scale factor can be used as a multiplier to find unknown lengths in similar shapes
- Understand that if shapes are similar, with a scale factor n, then their areas are also in proportion – scale factor n^2, and that their volumes are in proportion – scale factor n^3

You will need to think about:

What changes and what stays the same:

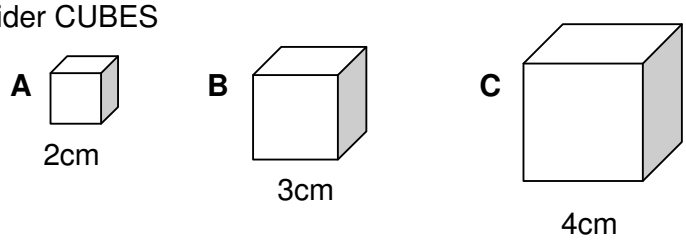
When enlarging a shape angle size remains unchanged, but lengths are increased by the scale factor.

Recognising the full requirements for enlargements:

Find or use the centre of enlargement
Find or use the scale factor
Understand that a fractional scale factor will reduce the size of a shape.

Be prepared to find and use enlargement scale factor in order to calculate lengths by using scale factors as multipliers and express the proportions as a ratio

e.g. Consider CUBES



Ratio A : B : C = 2 : 3 : 4

Length multipliers A to B = $\frac{3}{2}$ A to C = 2 (or $\frac{4}{2}$)

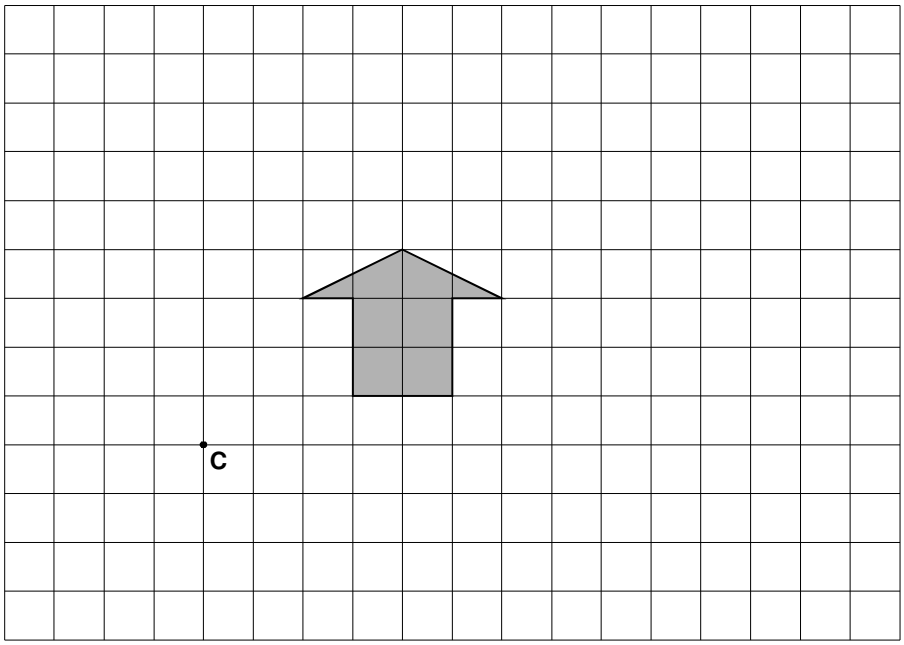
B to C = $\frac{4}{3}$ C to B = $\frac{3}{4}$

Quick Questions

1. The grid shows an arrow.

   On the grid, draw an **enlargement** of **scale factor 2** of the arrow.

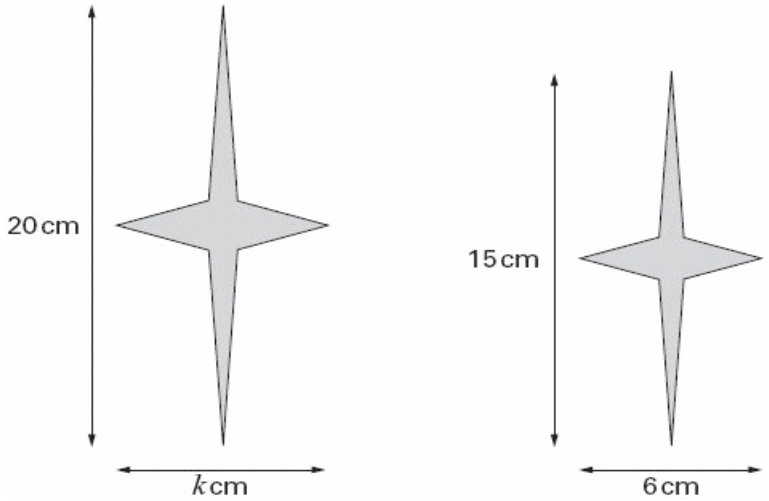
   Use **point C** as the centre of enlargement.



2. A rectangle has an area of eight square centimetres.
   What is the area of a rectangle with sides that are all twice as long?

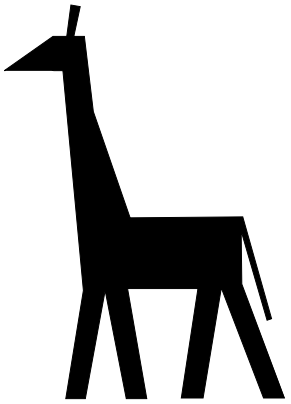
3. The scale on a map is one centimetre to five metres.
   On the map the length of a street is eight centimetres.
   What is the real length of the street in metres?

4. The diagram shows two shapes that are mathematically similar.



   What is the value of k?

5. Jill has drawn an original picture of a giraffe for an animal charity.
It measures 6.5cm high by 4cm wide.

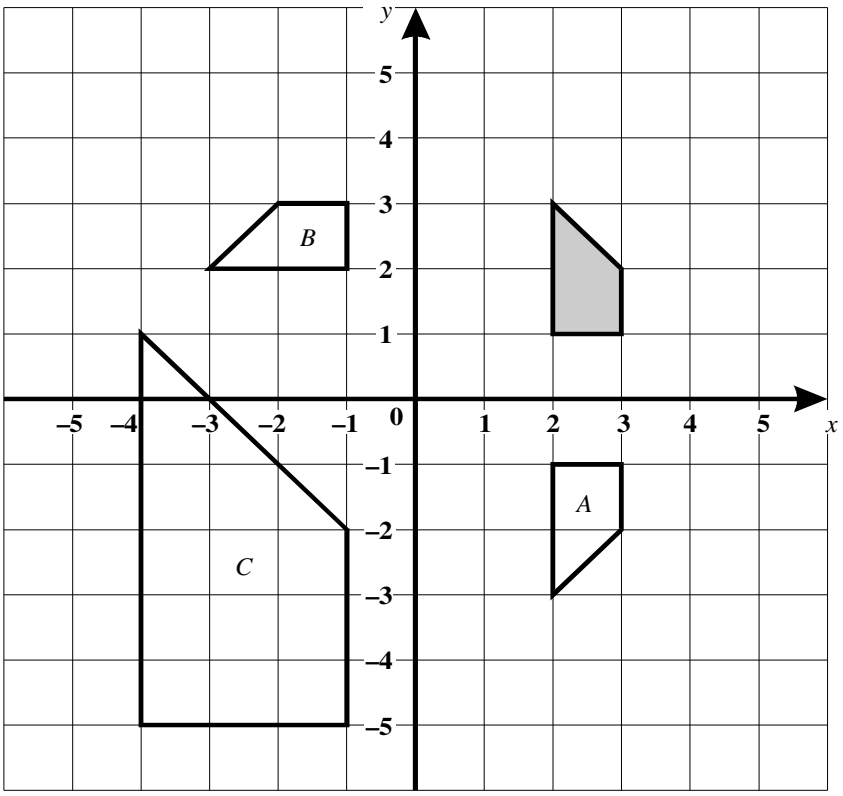


ORIGINAL PICTURE

Jill wants to enlarge the original picture so that it **just** fits inside a rectangle on a carrier bag.
The rectangle measures 24cm high by 12cm wide.

By what scale factor should she multiply the original picture?

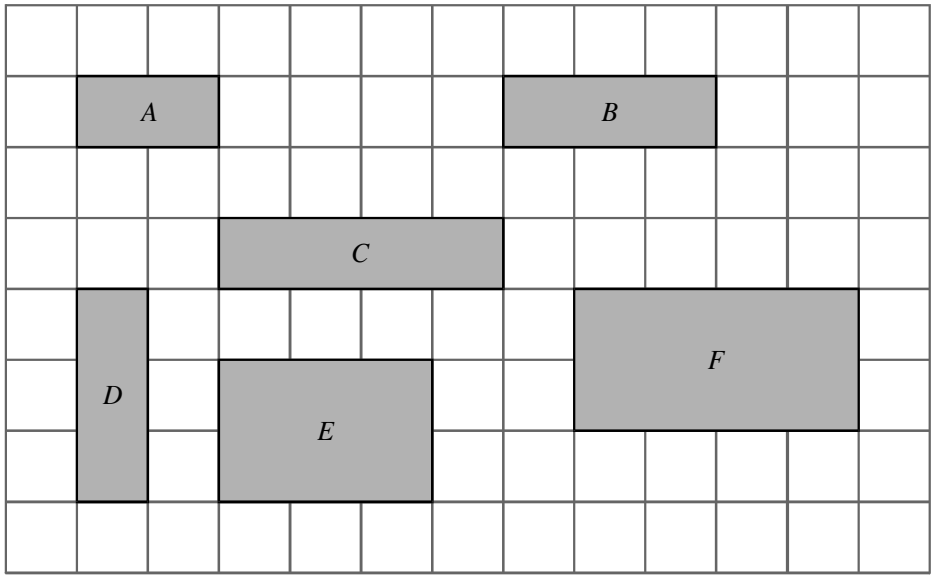
6.



Describe fully a single transformation that would map the shaded shape on to shape *C*.

Past Paper Questions *(From AQA GCSE Papers)*

1. Here are six rectangles on a centimetre grid.



(a) Which two rectangles are congruent?

Answer ..... and .....

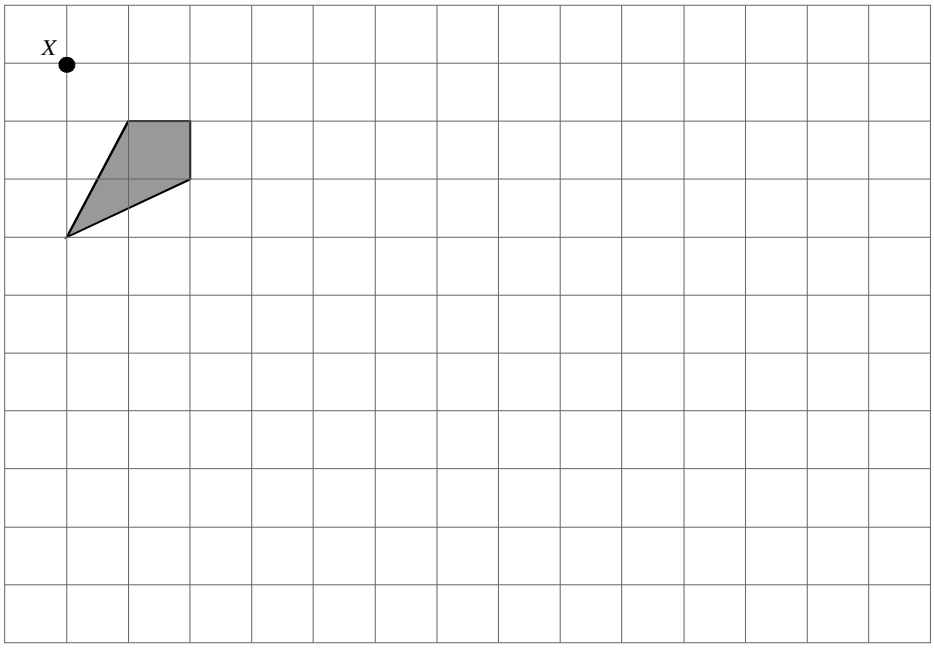
(1)

(b) Which two rectangles are similar?

Answer ..... and .....

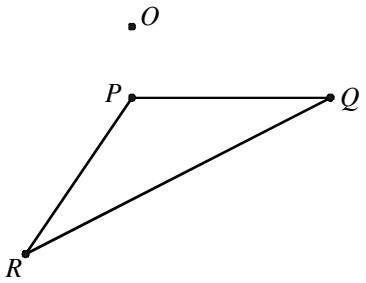
(1)

2. On the grid below, draw an enlargement of the kite, scale factor 2, centre *X*.



(2)

3. (a) Triangle PQR is mapped onto triangle $P_1Q_1R_1$, by an enlargement, centre O, scale factor 3. Draw $P_1Q_1R_1$.

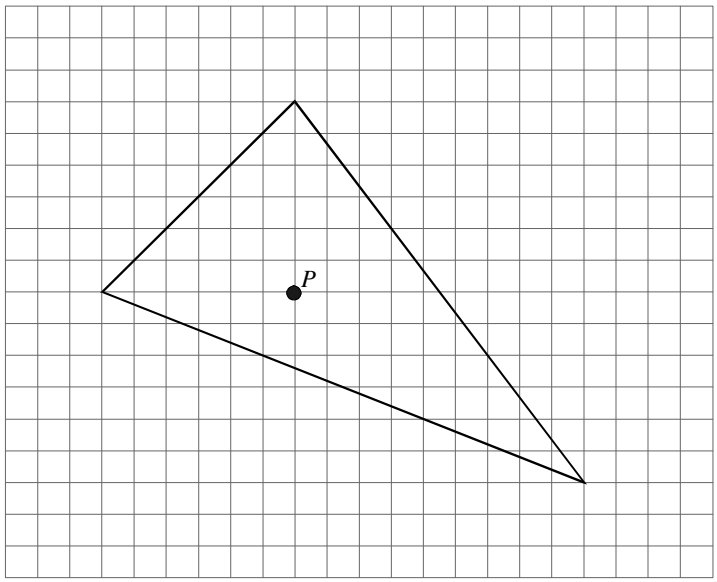


(3)

(b) Describe fully the single transformation which maps $P_1Q_1R_1$ onto PQR.

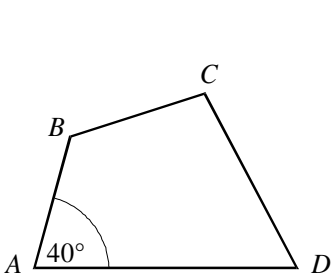
..... **.(2)**

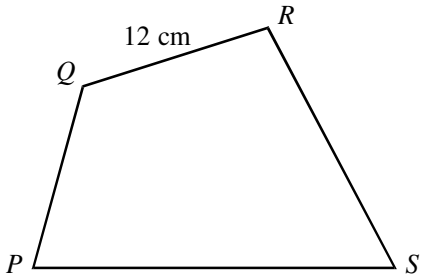
4. Enlarge the triangle with scale factor $\frac{1}{3}$ centre P.



(2)

5. *PQRS* is an enlargement with scale factor 1.5 of *ABCD.*





Not drawn accurately

(a) Calculate the length of *BC.*

.....

.....

Answer *BC* = ... cm

(2)

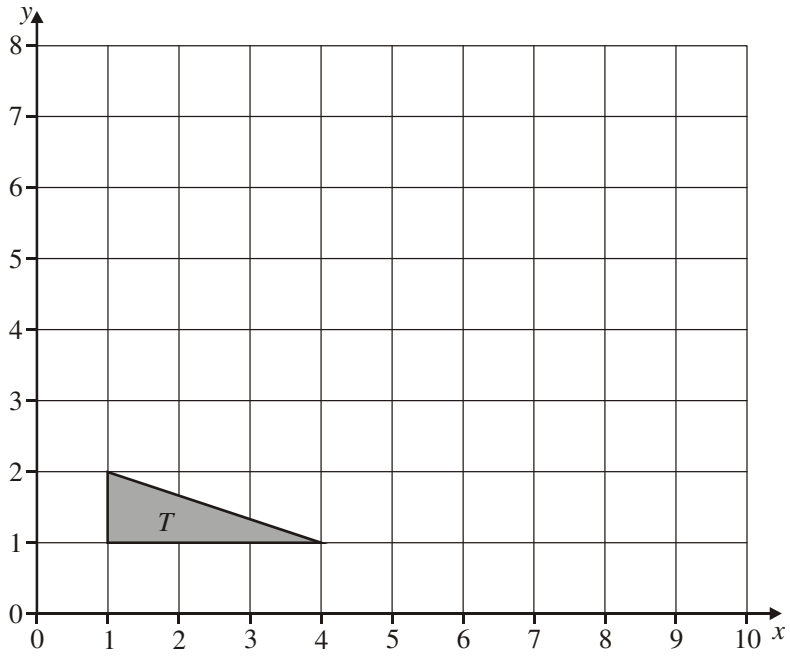
(b) Write down the size of angle *QPS*.

.....

Answer *QPS* = ... degrees

(1)

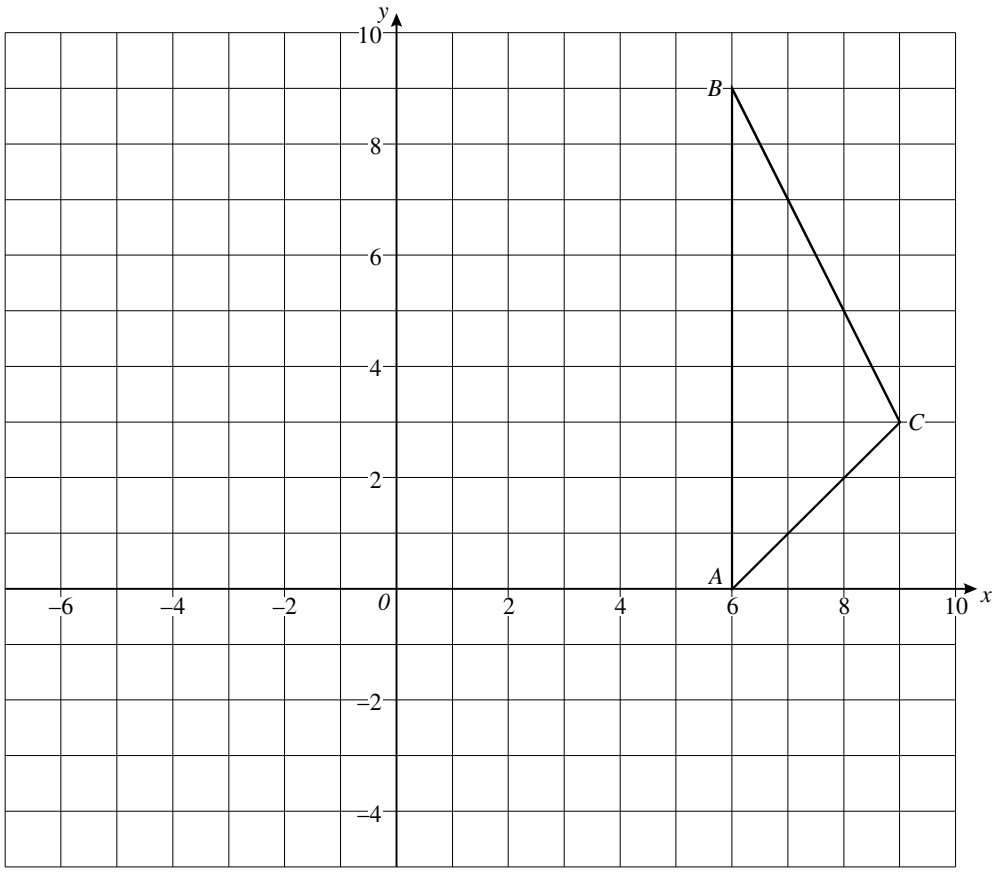
6. The vertices of triangle *T* are (1,1), (1,2) and (4,1).



Enlarge triangle *T* by scale factor 2, with (0,0) as the centre of enlargement.

(3)

7. Triangle *ABC* has vertices *A* (6, 0), *B* (6, 9), *C* (9, 3)



(a) Rotate triangle *ABC* through 180° about the point (2, 4)
Label the image triangle *R*.

(2)

(b) Enlarge triangle *ABC* by scale factor $\frac{1}{3}$ from the centre of enlargement (3, 0)
Label the image triangle *E*.

(2)

(c) Describe fully the **single** transformation which maps triangle *E* to triangle *R*.

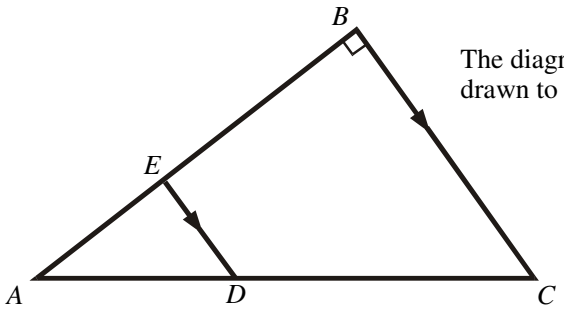
.....

.....

(3)

8.

ABC is a right-angled triangle.
Angle *ABC* = 90°.
ED is parallel to *BC*.
AB = 7.5 cm, *BC* = 4 cm.



The diagram is not drawn to scale.

(a) Calculate the length of *AC*.

.....

(3)

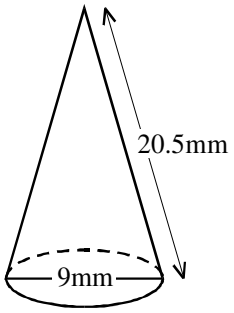
(b) Given that triangle ADE : triangle ACB is 1 : 5

Calculate the length of *AD*.

.....

(3)

9. A silver earring is in the shape of a solid cone, as shown in the diagram.
The slant height of the cone is 20.5 mm and the diameter of the base is 9 mm.



not drawn accurately

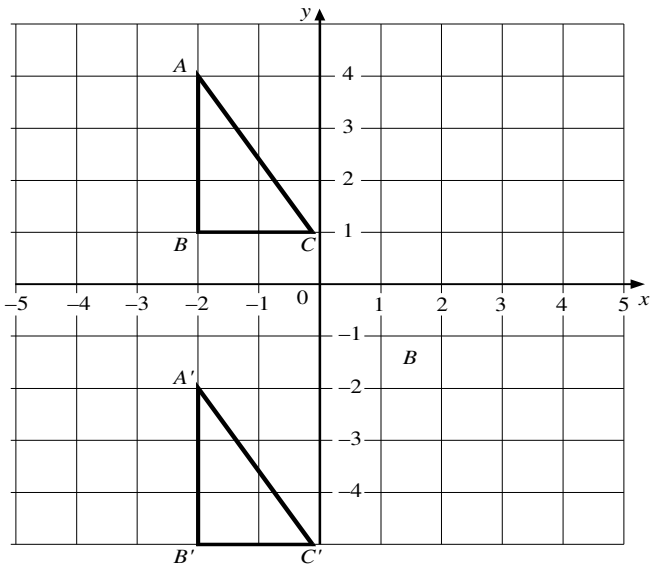
A similar solid cone of base diameter 13.5 mm is used as a pendant for a necklace. What would the slant height of the pendant's cone be?

……

……

(3)

10. The grid below shows a triangle *ABC* and a triangle *A'B'C'*.



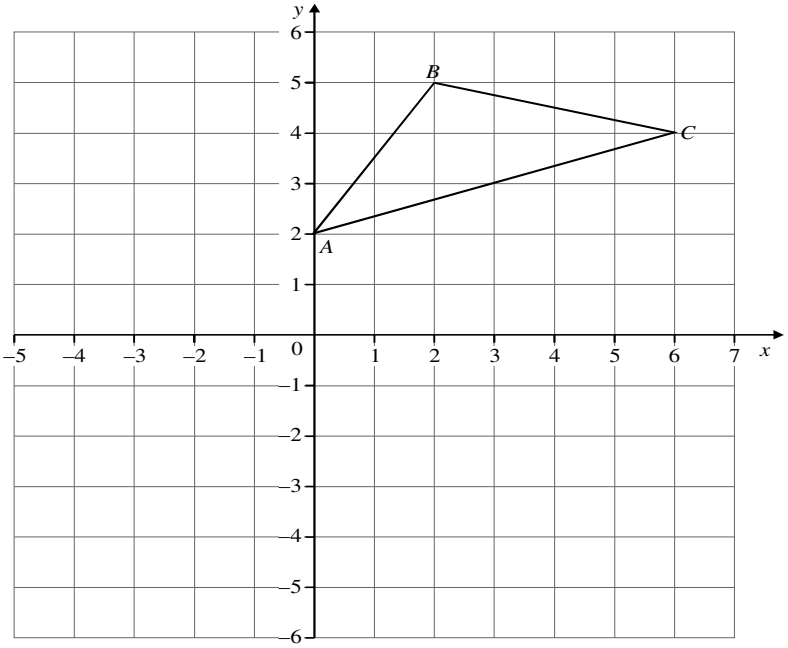
(a) Draw the triangle *A"B"C"* which is an enlargement of *ABC* with a scale factor ½ with centre (2, 1).

(2)

(b) Describe fully the transformation that takes triangle *A"B"C"* to triangle *A'B'C'*.

…… **(2)**

11. Triangle *ABC* has vertices at *A* (0, 2), *B* (2, 5), *C* (6, 4).



Draw the enlargement of triangle *ABC* with scale factor $\frac{1}{2}$ and centre (2, 2). **(2)**

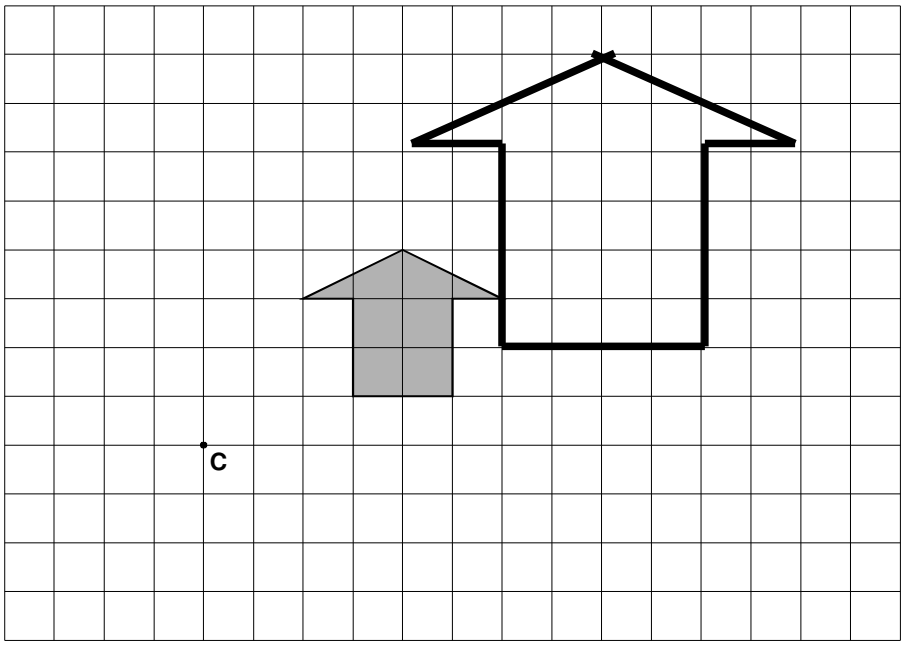
END OF QUESTIONS

30-4-10 Shape & Space Answers

Topic: Enlargement & Similarity

Answers to Quick Questions

1.



2. length doubled x 2 and width doubled x 2 gives 8 x 2 x 2 = 32 cm^2

3. 1 cm ≡ 5 m so 8 x 5 = 40 m

4. K = 4.5 cm

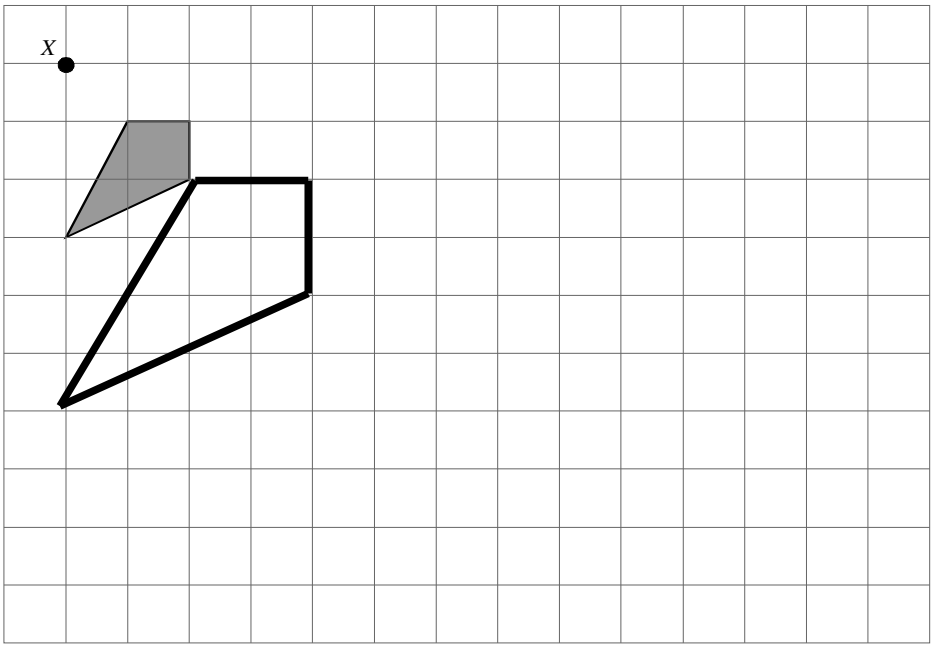
5. multiply by 3 (roughly)

6. Transform by enlarging from centre (5,4) with a scale factor of 3

Answers to Past Paper Questions

1. (a) rectangles B and D are congruent (identical)
(b) rectangles A and F are similar (F is A x 2)

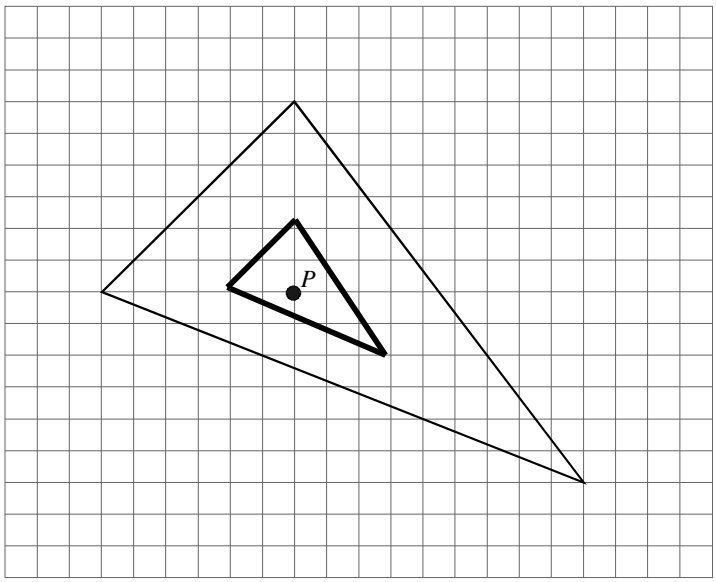
2.



3. (a) Construct with straight lines fro. Centre O, thru' P, R and Q to produce a shape with lengths 3 times as long.

(b) Enlargement from centre O with scale factor 1/3

4.

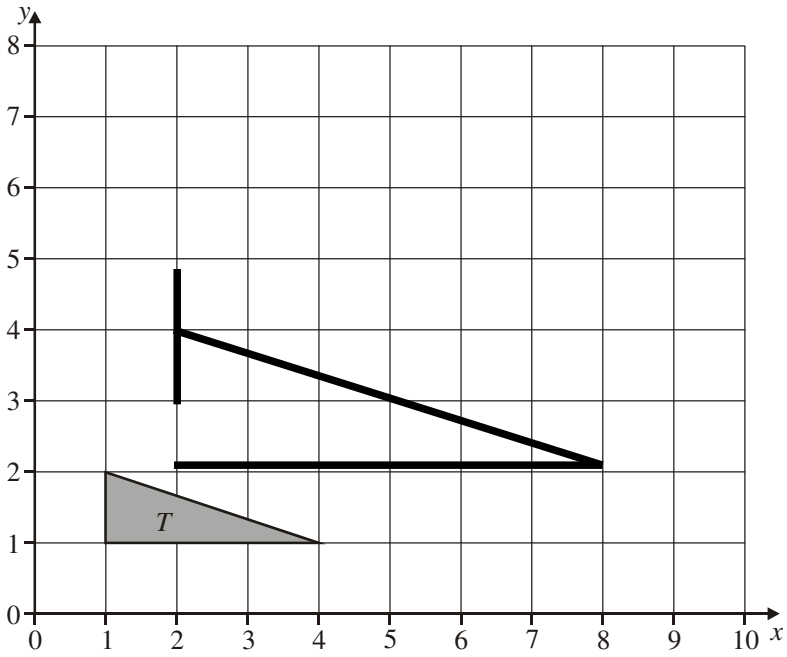


5. (a) ABCD to PQRS is x $\frac{3}{2}$ so reversing needs x $\frac{2}{3}$

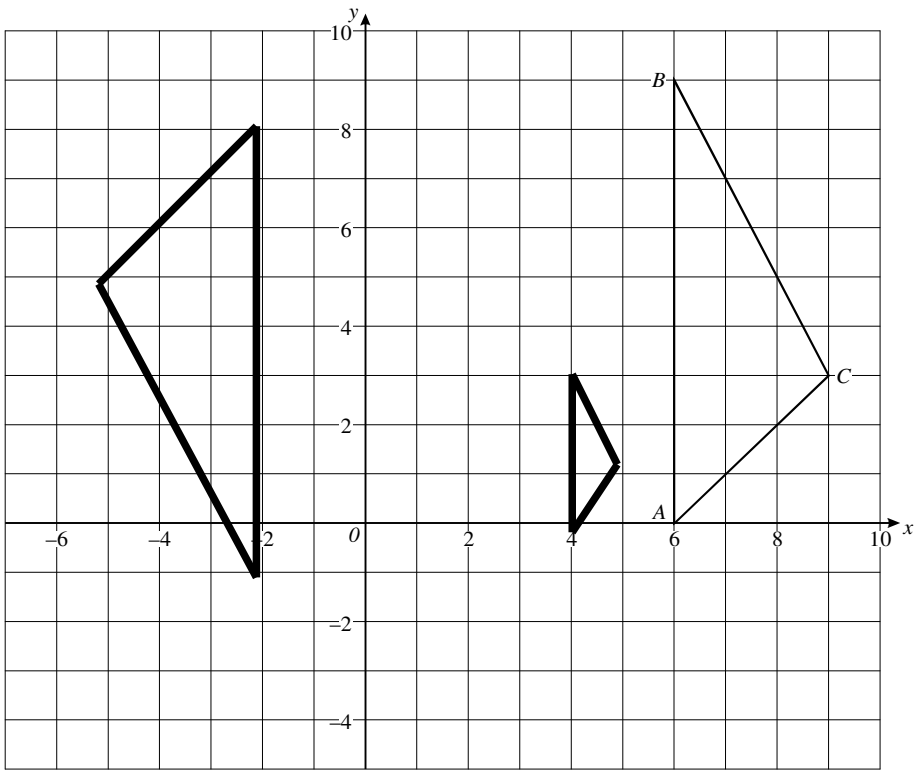
So 12 x $\frac{2}{3}$ = 8 cm

(b) angle QPS = 40 (as angles are not changed by enlargement)

6.



7.



(c) E is enlarged buy scale factor **- 3** through centre (2,2)

8. (a) By Pythagoras Rule: $AC^2 = 7.5^2 + 4^2 = 72.25$

So AC = $\sqrt{72.25}$ = 8.5 cm

(b) AD is 1/5 of AC so 8.5 ÷ 5 = 1.7 cm

9. The enlargement scale factor will be $\frac{13.5}{9}$

So slant height will be 20.5 x $\frac{13.5}{9}$ = 30.75 mm

10. (a) A'' = (0,2.5) B'' = (0, 1) C'' = (1,1)

(b) Enlargement scale factor 2 centre (2,6)

11. Coordinates of the enlarged shape should be (1, 2) (2, 3.5) (4, 3)

END OF ANSWERS

30- 4-10 Shape and Space
DRAFT

Enabling world-class education

Topic: Geometrical Reasoning 1 – Angle Properties

You need to be able to:

- Understand and use angle measure
- Understand and use angle facts related to triangles and quadrilaterals
- Understand and use angle facts related to parallel lines

You will need to think about:

Which angle facts are relevant to the problem you want to solve.

Some of the most common ones you need to know are:

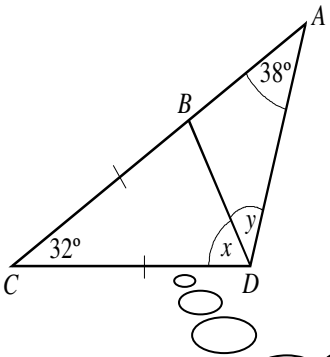
- Angles around a point add up to 360º
- Angles in a triangle add up to 180º
- Angles in a quadrilateral add up to 360º
- Alternate angles on parallel lines are equal
- Corresponding angles on parallel lines are equal
- Vertically opposite angles are equal

You also need to think about which other properties of polygons may be helpful. These include properties of regular shapes, special triangles, quadrilaterals and other polygons.

e.g An equilateral triangle has all sides equal and all angles equal to 60º
An isosceles triangle has two equal sides and two equal angles

Properties of polygons are covered in more detail in the next unit.

To find x and y in the diagram below, think of which of the above facts you may need:



The diagram is not drawn to scale.

Triangles are involved so you probably need to use the fact that angles in a triangle add up to 180º

CB and CD are marked as equal, you will need to use this fact

One way of solving this problem is shown below:

Find *x* first
In triangle CBD CB=CD (the triangle is isosceles)
The angles in triangle CBD add up to 180º
180-32 = 148º
x = ½ of 148º = 74º (angle CBD = angle CDB as they are equal angles in an isosceles triangle)

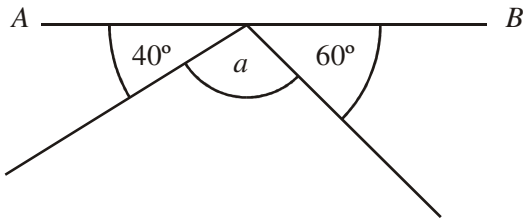
Now use the big triangle ACD to find y
The angles add up to 180º
So 32 + 38 + x + y = 180
x=74º
70 + 74 + y = 180
144 + y = 180
Y = 180 – 144 = 36º

(Often there is more than one way to solve the problem!)

Quick Questions

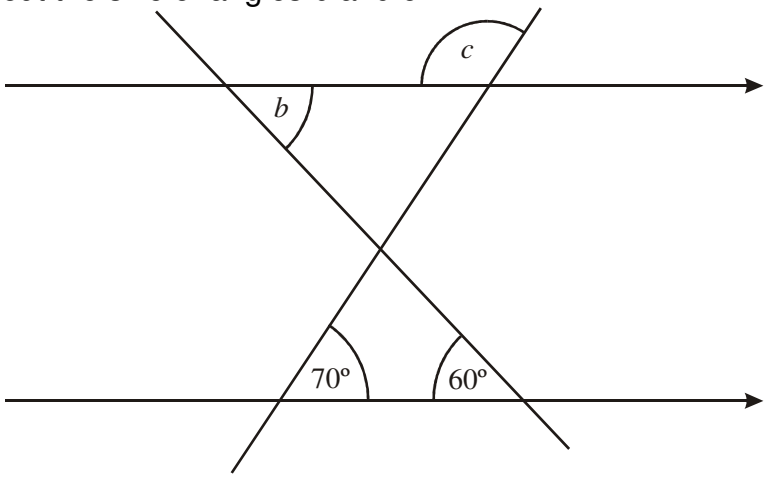
1. Two angles in a triangle are 36º and 80 º.
   What is the third angle in the triangle?

2.
Work out the size of angle a



Not drawn accurately

3. Work out the size of angles b and c



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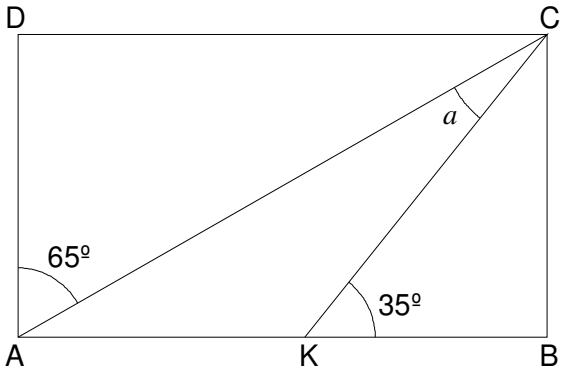
4. Triangle ABC is isosceles with AB = BC and angle ABC = 120º
What is the value of angle BAC? (a sketch will help)

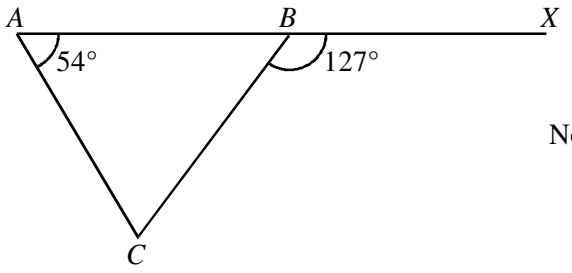
5. In the diagram below, what is the size of angle a?



Not drawn accurately

Past Paper Questions *(From AQA GCSE papers)*

1. (a) In the diagram, *ABX* is a straight line.



Not to scale

Work out the size of angle *ACB.*

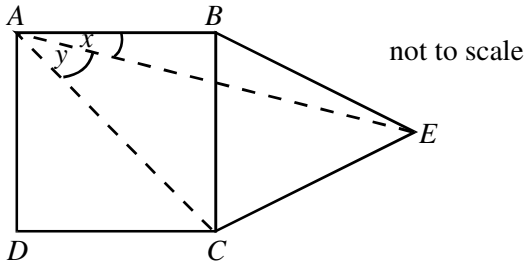
.....

.....

Answer ..... degrees

(2)

2.



not to scale

ABCD is a square. *BEC* is an equilateral triangle.

(a) Write down the size of

(i) angle *ABC*, ...

(1)

(ii) angle *EBC*. .....

(1)

(b) Calculate the size of

(i) the angle marked *x,*

.....

.....

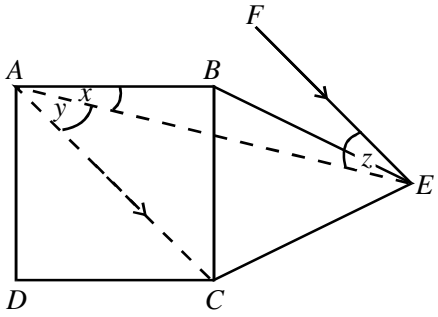
(1)

(ii) the angle marked *y,*

.....

(1)

(c)



not to scale

AC is parallel to *FE*.

(i) What is the size of the angle marked *z*?

.....

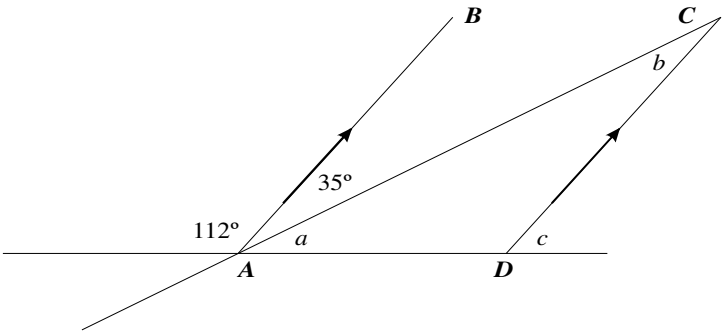
(1)

(ii) Give a reason for your answer.

.....

(1)

3. In the diagram AB is parallel to *CD*.



In each case give the size of the angle.

(i) Angle *a*

.....

(1)

(ii) Angle *b*.

.....

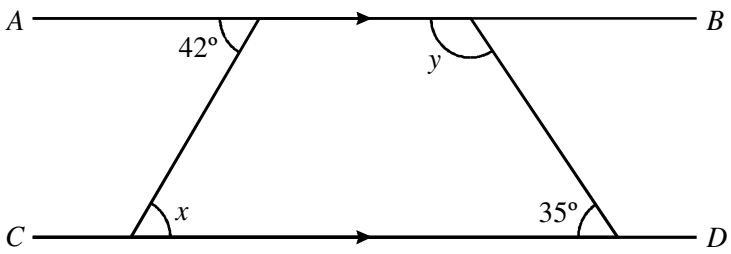
(1)

(iii) Angle *c*.

.....

(2)

4.



Not drawn accurately

AB is parallel to *CD.*

(a) Write down the size of the angle marked *x*.

.....

(1)

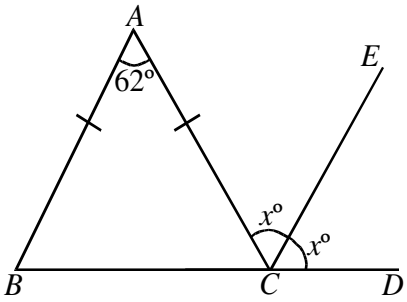
(b) Calculate the size of the angle marked *y*.

.....

.....

(1)

5.



The diagram is not drawn to scale.

BCD is a straight line. *CE* bisects angle *ACD*.

AB = *AC*.

(a) Calculate the value of *x*.

.....

.....

(3)

(b) Is *EC* parallel to *AB*?
Give a reason for your answer.

.....

(1)

Shape and Space 1

Answers to Quick Questions

1. Angles in a triangle add up to 180 º

80 + 36 = 116

180 – 116 = 64º

2. Angles on a straight line add up to 180 º

60 + 40 = 100

180-100 = 80 º

3. b = 60 º (alternate angles on parallel lines)

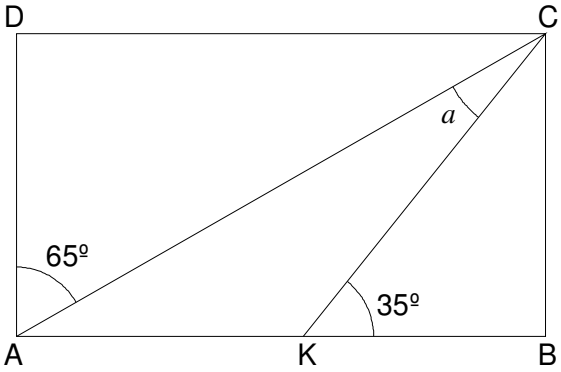
c= 180 -70 = 110 º

(using alternate angle followed by angles on straight line add up to 180 º)

4. 180 – 120 = 60 º

Angle BAC = ½ of 60 = 30 º

5.



Not drawn accurately

In triangle ACK
Angle CAK = 90 – 65 = 25º
Angle AKC = 145º (angles on a straight line add up to 180º)
Angles in a triangle add up to 180º
a= 180 – (25 +145) = 180 – 170
a = 10º

Answers to past paper questions

1. Angle ABC = 180 – 127 = 53 º (angles on a straight line add up to 180 º)
   Angle ACB = 180 – 54 – 53 = 73 º (angles in a triangle add up to 180 º)

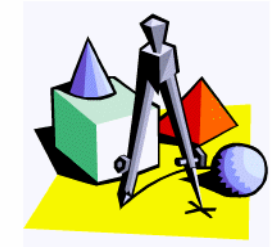
2 a) (i) angle ABC = 90º (corner of a square)
(ii) angle EBC = 60 º (angles in an equilateral triangle are equal)
b) (i) AB=BE (sides of an isosceles triangle)
angle ABE = 90 +60 = 150 º
x = ½ of (180-150) = 15º
(ii) angle BAC = 45 º
y = 45-15 = 30 º
c) (i) angle z = angle y = 30 º
(ii) alternate angles on parallel lines

3
a = 180 – 112 – 35 = 33º (angles on a straight line add up to 180 º)
b = 35 º (alternate angles on parallel lines)
c= 35 + 33 = 68 º (corresponding angles on parallel lines)

4. x = 42º (alternate angles on parallel lines)
   y = 180 – 35 = 145º

5.
(a) Angle ACB = (180 – 62) ÷ 2 = 59º
x = (180 – 59) ÷ 2 = 60.5º

(b) EC is **not** parallel to AB.
If it was angle ACB would be equal to x (alternate angles on parallel lines are equal)
This is not true as seen in the calculation above.



30-4-10 Shape and Space
DRAFT

Topic: Geometric Reasoning 2 - Properties of Shapes

You need to:

- Recognise shapes and their mathematical names
- Know properties of common shapes
- Understand and use angle facts related to triangles, quadrilaterals and other polygons.
- Understand and use angle facts related to parallel lines

You will need to think about:

Which shapes are involved and which facts are relevant to the problem you want to solve. You will need to know the angle properties covered

Special Triangles including:
Isosceles Triangles – two equal sides and two equal angles
Equilateral Triangle – three sides equal and all angles equal to 60 º

Special Quadrilaterals:
Square – all sides equal and angles equal to 90º. Diagonals bisect each other at right angles.
Rectangle – opposite sides equal and angles equal to 90 º; diagonals equal
Parallelogram – opposite sides equal and parallel; opposite angles equal
Rhombus – all sides equal; opposite sides parallel; opposite angles equal; Diagonals bisect each other at right angles.
Trapezium – one pair of parallel sides
Kite – pairs of adjacent sides equal. Diagonals intersect at right angles. One pair of opposite angles equal.

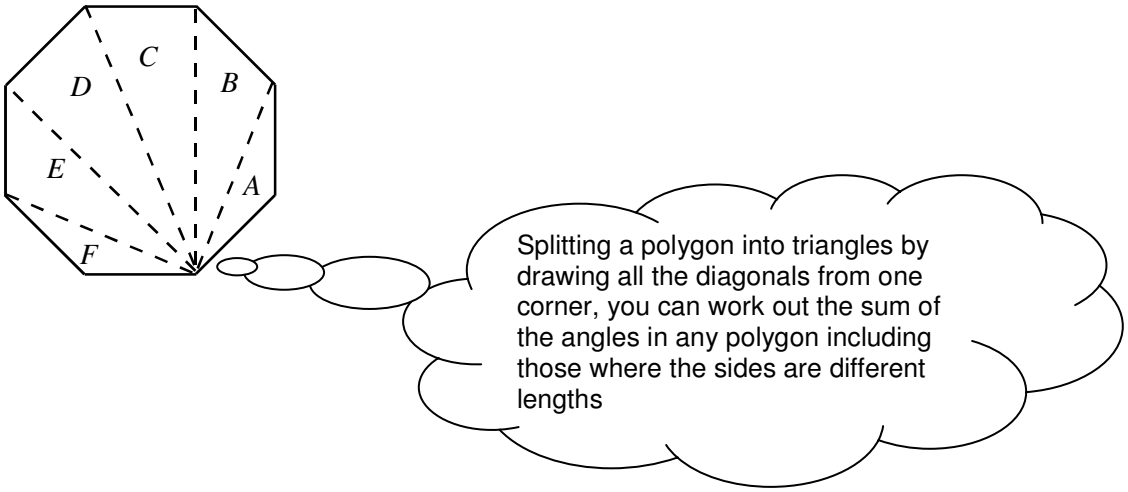
In solving problems involving shapes, you will need to use basic angle properties together with properties of the shape involved

Other Polygons
Pentagon – 5 sides
Hexagon – 6 sides
Heptagon – 7 sides
Octagon – 8 sides
Nonagon – 9 sides
Decagon – 10 sides
In a regular polygon all sides and angles are equal

You also need to think about symmetry properties of polygons

Shapes which are identical are **congruent** to each other

The diagram below shows a regular polygon (its sides are all equal and its interior angles are equal)

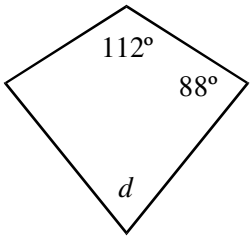


Sum of angles in each of the triangles A,B,C,D,E and F = 180 º
Sum of angles in **any** octagon = 6 x 180 = 1080º

As the octagon shown here is **regular.**
Each interior angle = 1080 ÷ 8 = 135º

Quick Questions

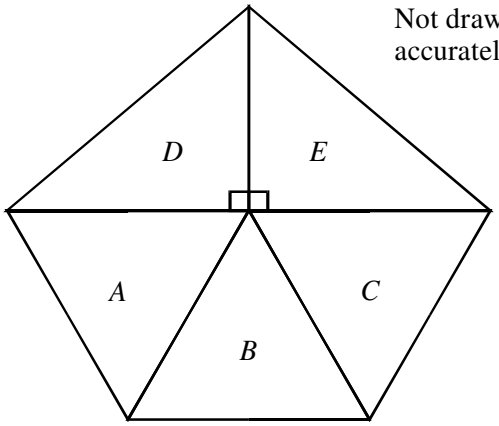
1. Write down the name of a shape with 6 equal sides
2. Name a shape with exactly one pair of parallel sides.
3. Name a shape with three lines of symmetry and rotational symmetry order 3
4. A shape ABCD has four equal sides but no right angles. What is the name of the shape ABCD?
5. Name a quadrilateral with exactly two lines of symmetry.
6. Draw a pentagon. Calculate the sum of the interior angles of the pentagon.
7. What is the size of angle d in the kite shown below?



8. Decide whether these statements are True or False
   (a) All squares are rectangles
   (b) A parallelogram has two pairs of parallel sides
   (c) Opposite angles in a kite are equal
   (d) A parallelogram has one line of symmetry

Past Exam Questions *(From AQA GCSE papers)*

1. This is a shape with five sides.
It is made from 5 triangles *A*, *B*, *C*, *D* and *E*.



(a) What is the mathematical name for a shape with five sides?

……

(1)

(b) Triangles *A*, *B* and *C* are all equilateral triangles.
What is special about the angles in equilateral triangles?

……

……

(1)

(c) Triangles *D* and *E* are congruent.
What does the word congruent mean?

……

……

(1)

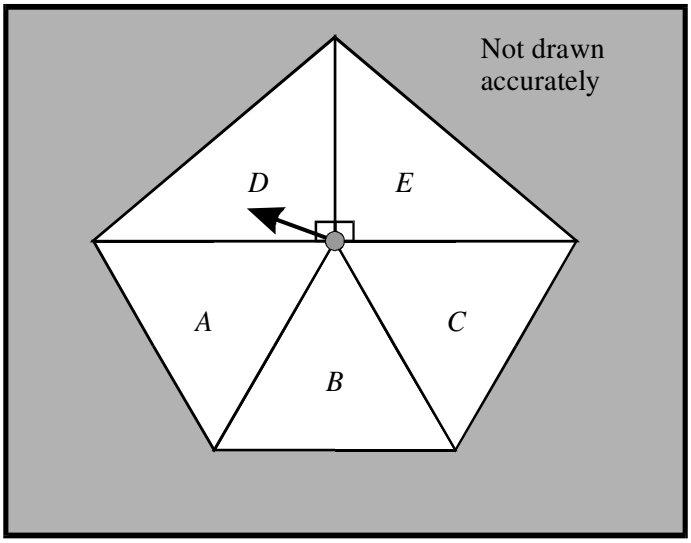
(d) One of the angles in triangle *D* is 30°
Write down the sizes of the other 2 angles.

……

……

(2)

(e) The shape is placed on a table and an arrow is fastened to make a spinner for a game.



The arrow is spun.

(i) Is the arrow equally likely to land on any triangle?
Give a reason for your answer.

……..

……..

(1)

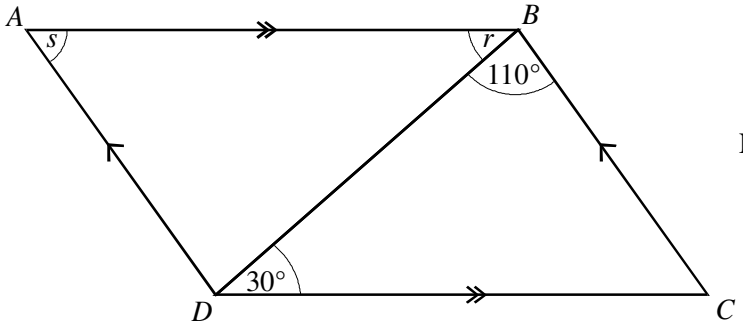
(ii) Calculate the probability that the arrow will stop on triangle *D*.

……..

……..

(2)

2. A parallelogram *ABCD* is shown.



Not to scale

DB is a diagonal. Angle *BDC* = 30°. Angle *DBC* = 110°.

(a) What is the size of angle *r*?

.....

Answer ..... degrees

(1)

(b) What is the size of angle s?

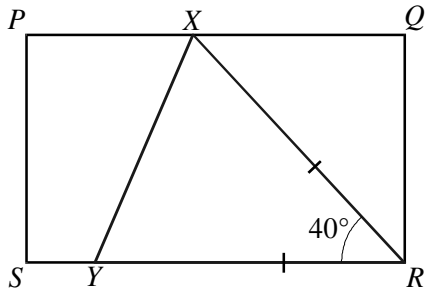
.....

.....

Answer ..... degrees

(2)

3. (a) The diagram shows a rectangle $PQRS$ with X on PQ and Y on RS.



Not to scale

(i) Which of the following correctly describes the quadrilateral $PXYS$?

Rhombus **parallelogram** **trapezium** **kite** **oblong**

Answer .....

(1)

(ii) $RX = RY$ and angle $XRY = 40°$.

Work out the size of angle QXR and angle XYS.

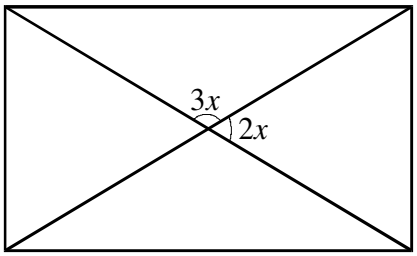
.....

Answer Angle QXR = ... degrees

Angle XYS = ... degrees

(3)

(b) The diagram shows another rectangle.



Not to scale

Work out the value of x.

.....

.....

Answer x = ... degrees

(2)

4. A quadrilateral with 4 equal sides and 4 right angles is called a square.

What is the mathematical name given to:

(a) A quadrilateral with 4 equal sides but no right angles?

.....

(1)

(b) A quadrilateral with 2 pairs of opposite sides equal but diagonals of different lengths?

.....

(1)

(c) A quadrilateral with only 1 pair of parallel sides of unequal lengths?

.....

(1)

5. Here is a list of quadrilaterals.

kite rectangle rhombus square trapezium

For each of the following descriptions, choose the correct name from the list.
You may find it helpful to sketch the quadrilaterals in the spaces provided.

(a) One pair of sides are parallel.
The other two sides are not parallel.

Answer .....

(1)

(b) All the angles are the same size.
Only opposite sides are equal.

Answer .....

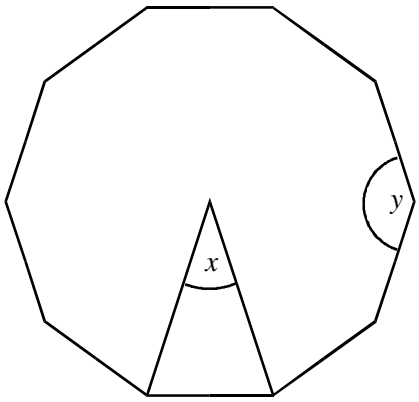
(1)

(c) All the sides are the same length. The diagonals are not equal in length.

Answer .....

(1)

6. The diagram shows a regular decagon.



Not drawn accurately

(a) Work out the angle at the centre of the decagon, marked x on the diagram.

.....

.....

Answer ..... degrees

(2)

(b) Work out the size of the interior angle, marked y on the diagram.

.....

.....

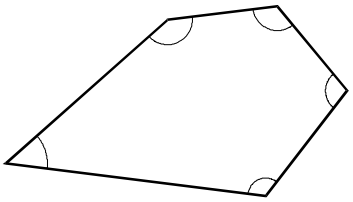
Answer ..... degrees

(2)

Shape and Space 2

Answers to quick questions

1. Regular hexagon
2. Trapezium
3. Equilateral triangle
4. Rhombus
5. Rectangle or rhombus (not a square)
6.



Draw in diagonals from one corner to create 3 triangles.
Sum of angles in a pentagon = 3 x 180 = 540º

7. 88 +88 + 112 = 288 º
360 - 288 = 72 º (angles in a quadrilateral add up to 360 º)

8. (a) True. All squares are rectangles with equal sides.
(b) True
(c) False. A kite only has one pair of opposite angles equal.
(d) False. A parallelogram has no lines of symmetry.

Answers to Past Exam Questions

1. (a) Pentagon

(b) All angles are same.

(c) Exactly the same.

.

(d) 90°
60°

(e) (i) No <u>AND</u> a reason which indicates that the angles at the centre are different.

<u>e.g</u>
Angles are not equal.
D is 90° but A is smaller.
D has more space at the spinner than A.
D is bigger than A.

(ii) 90/360 OR 1/4 or equivalent fraction.

2. (a) 30º (alternate angles on parallel lines)

(b) $180 - 110 - 30 = 40^\circ$

3. (a) (i) Trapezium

(ii) Angle $QXR = 40^\circ$ (alternate angles)

$Angle\ XYR = \frac{(180 - 40)}{2} = 70^\circ$

$Angle\ XYS = 110^\circ$

(b) $5x = 180$

$x = 36^\circ$

4. (a) Rhombus

Not diamond

(b) Parallelogram, rhombus

(c) Trapezium

5. (a) Trapezium

(b) Rectangle

(c) Rhombus

6. (a) $x = 360 \div 10 = 36^\circ$

(b) To calculate the interior angle of a decagon:
Draw all the diagonals from one corner
This gives 8 triangles
Sum of angles in the decagon = $8 \times 180 = 1440^\circ$

$y = 1440 \div 10 = 144^\circ$

Enabling world-class education

30-4-10 Shape and Space
DRAFT

Topic: Transformations & Coordinates

You need to be able to:

- Find the coordinates of a mid point of a line
- Recognise and transform 2D shapes by translation, reflection, rotation
- Identify reflection and reflective symmetry in 3D shapes
- Know the equations of common straight lines on a graph
- Transform 2D shapes with any *combination* of the above transformations

You will need to think about:

What changes and what stays the same:

When translating, reflecting or rotating a shape anjgle size, lengths and shape remain unchanged. The only change is that of position

Recognising the full requirements for describing or transforming by:

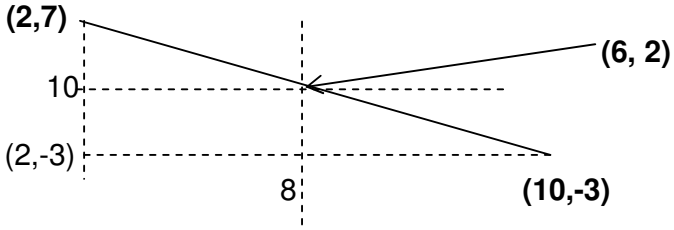
- Translation: column vector describes the movement

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

- Reflection: equation of the mirror or straight line
- Rotation: coordinates of the centre of rotation, direction and angle of rotation

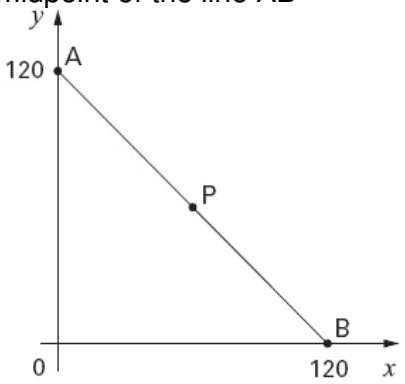
Use a sketch to help with finding coordinates:

e.g. mid point?

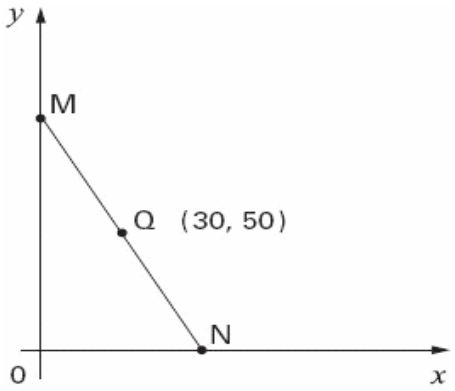


Quick Questions

1. Find the coordinates of P, the midpoint of the line AB

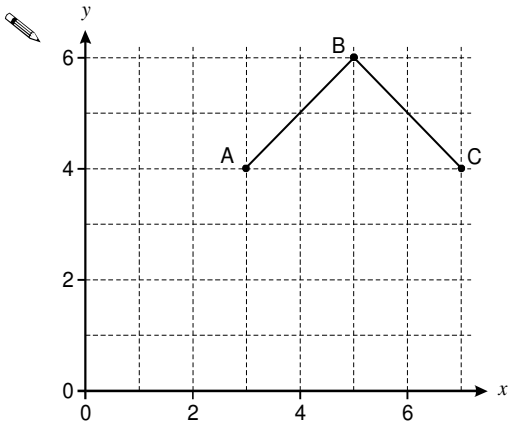


2.



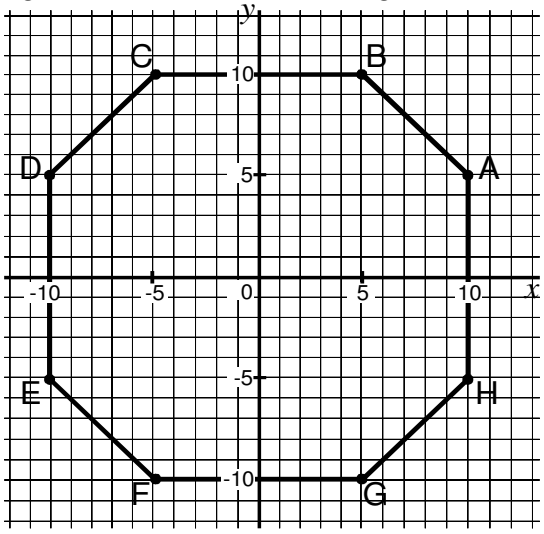
Find the coordinates of M, and N

3. (a) If ABCD is a square, what are the coordinates of D?



(b) If ABCDE ia a trapezium what are the coordinates of E?

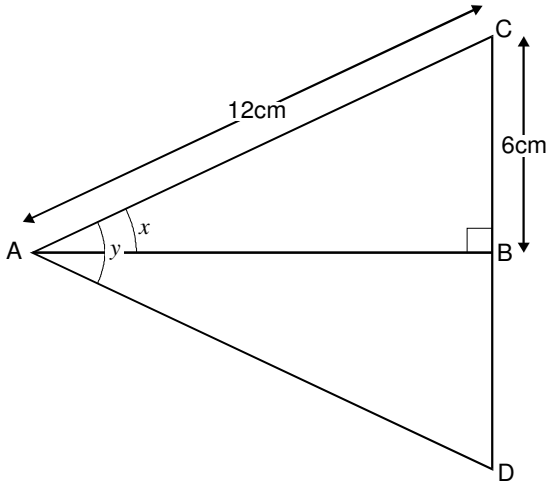
4. The diagram below shows an octagon ABCDEFGH



Write down the equations of the lines through

AH, DE, BC, EA, AB

5. Line AB is a mirror line in which triangle ABC is reflected to give ABD



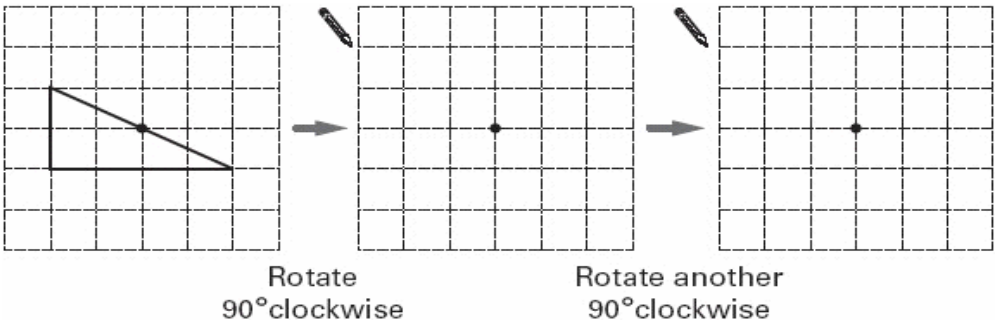
Not drawn accurately

Find the lengths AD and DC

Explain why ACD is an equilateral triangle

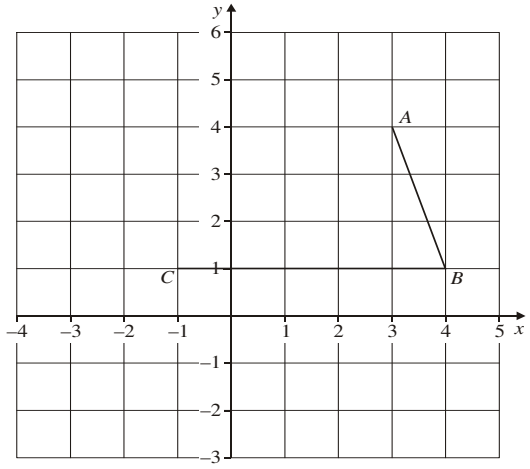
Find the angles x and y

6.



Past Paper Questions *(From AQA GCSE papers)*

1. Two sides of a parallelogram are drawn on the grid below.



(a) Write down the coordinates of the point *A*.

Answer (..... , ...)

(1)

(b) Write down the coordinates of the point *C*.

Answer (..... , ...)

(1)

(c) (i) Draw two more lines to complete the parallelogram *ABCD*.

(1)

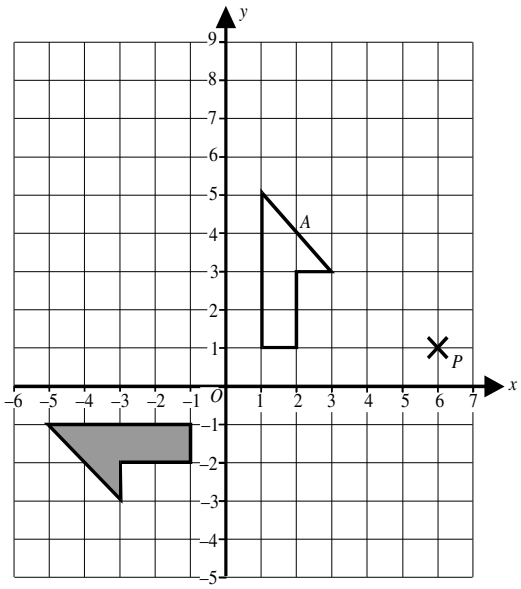
(ii) Write down the coordinates of *D*.

Answer (..... , ...)

(1)

(Total 4 marks)

2.



(a) Describe fully the single transformation that will transform the shape labelled *A* to the shaded shape.

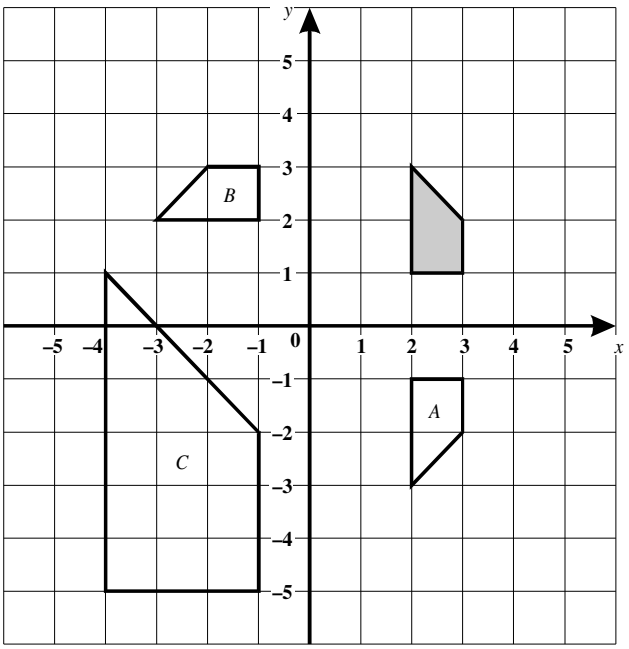
.....
.

(2)

(b) On the grid draw the shape labelled *A* after it has been rotated 90□ clockwise about the origin. Label it *B*.

(2)

3.



Describe fully a single transformation that would map the shaded shape on to

(a) shape *A*,

.....

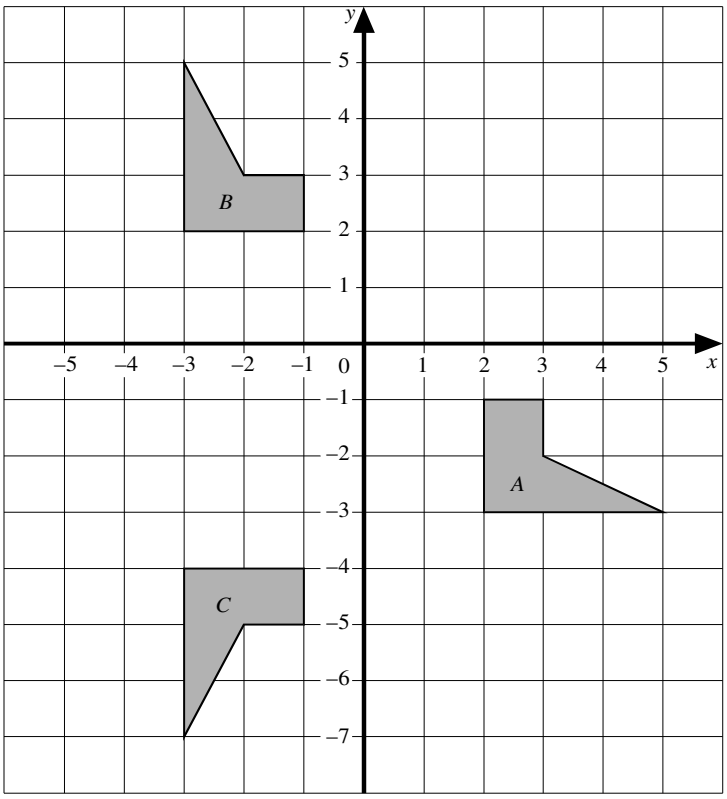
(2)

(b) shape *B*,

.....

(2)

4.



(a) Describe the single transformation that will transform shape *C* to shape *B*.

.....

(2)

(b) Describe the single transformation that will transform shape *A* to shape *C*.

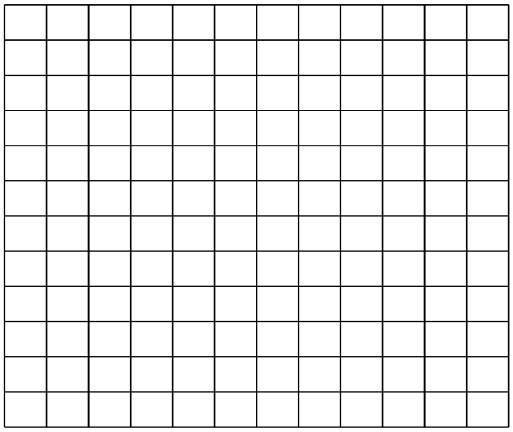
.....

.....

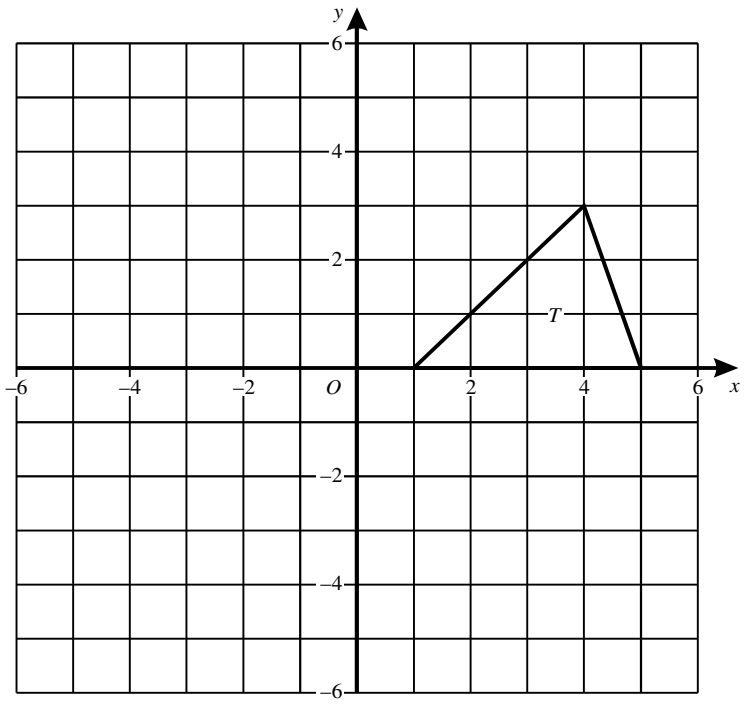
(3)

5. Find the coordinates of the reflection of the point (1, 4) in the line $y = -x$
(You may find the grid below useful.)

.....



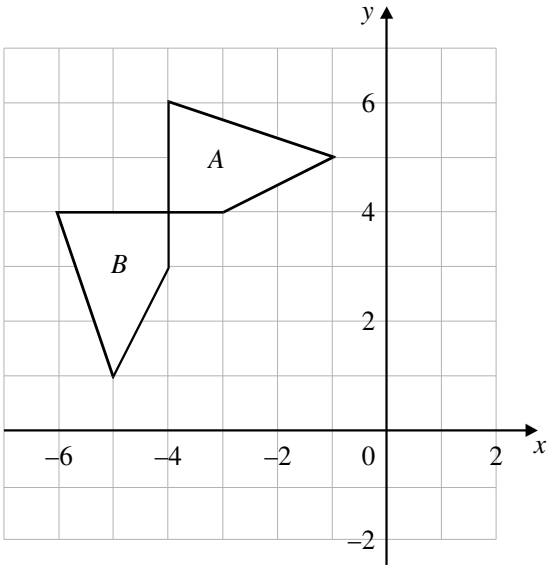
6. The triangle, labelled *T*, has vertices (5, 0), (1, 0) and (4, 3).



Triangle *T* is reflected in the line $y = x$.

Draw the new triangle. Label it *A*.

7. (a)

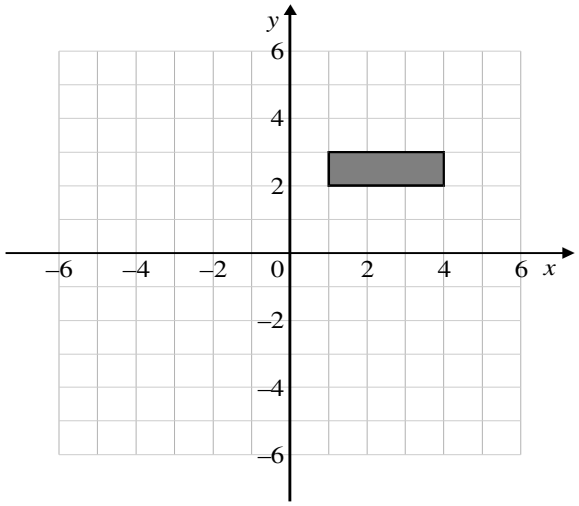


Describe fully the transformation which maps shape A onto shape B.

.....

(2)

(b)



The rectangle is rotated 90' clockwise about the point ~(1, 0) and then translated by vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

Draw the final position of the rectangle after these transformations.

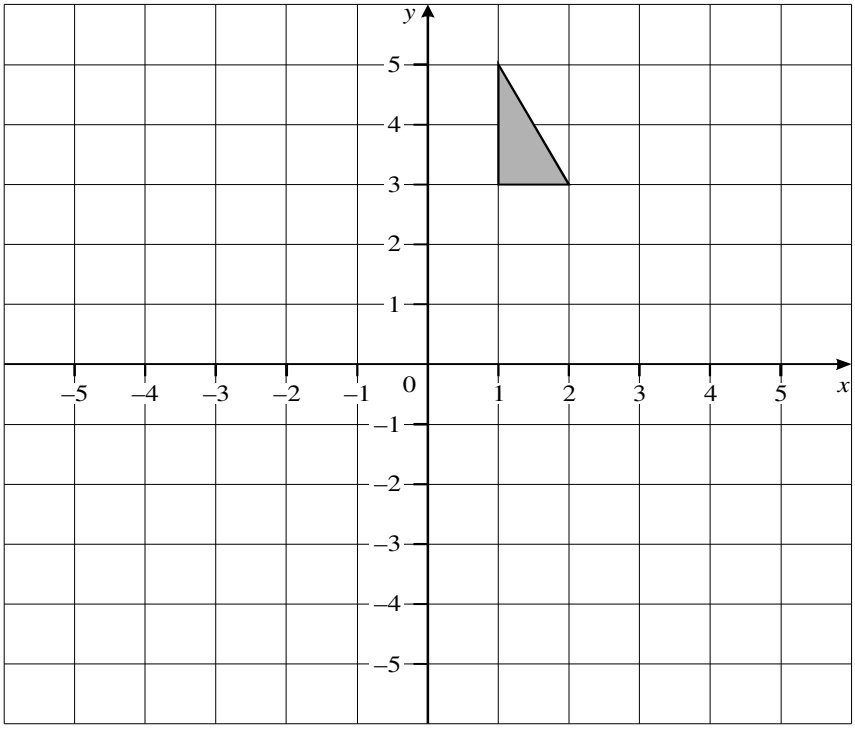
(4)

8. (a) On the grid below

(i) reflect the shaded triangle in the line $y = x$.
Label it *A*. **(2)**

(ii) rotate the shaded triangle 90° anti-clockwise about (1, 1).
Label it *B*.

(2)

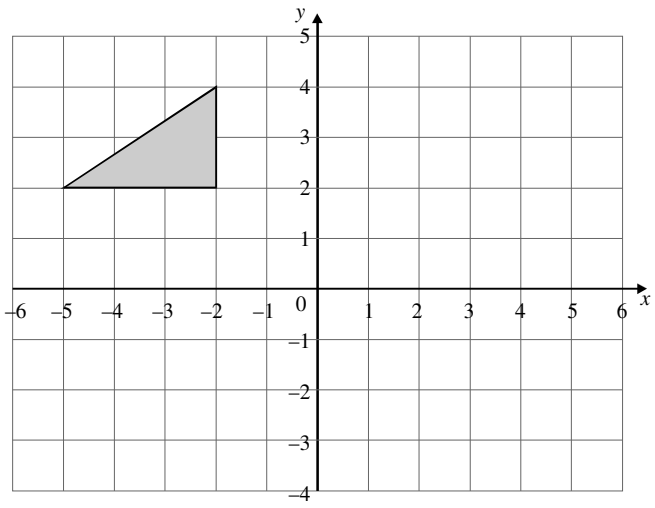


(b) Describe the **single** transformation that takes triangle *A* to triangle *B*.

.....

(2)

9.



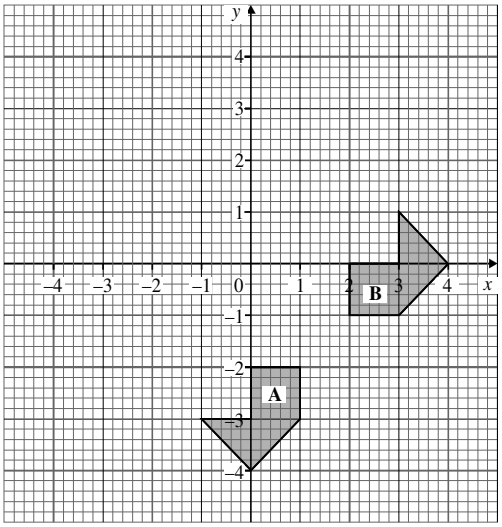
(a) Translate the shaded triangle by the vector $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$ Label this triangle C.

(2)

(b) Rotate the shaded triangle through 90° clockwise about (1, 1) Label this triangle D.

(2)

10. The diagram shows two positions of a shape.



(a) Describe fully the single transformation which maps *A* onto *B*.

.....

(2)

(b) *A* is mapped onto *C* by a translation with vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Draw the position of *C* on the diagram. **(2)**

END OF QUESTIONS

30-4-10 Shape & Space Answers

Topic: Transformations & Coordinates

Answers to Quick Questions

1. P = (60,60)

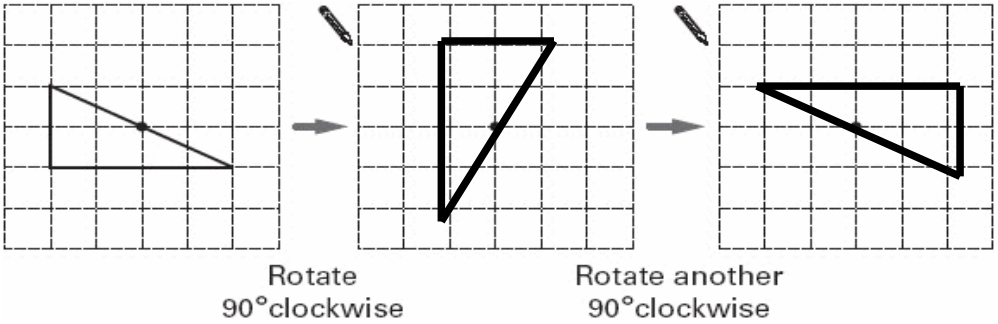
2. M = (0, 100) N = (60,0)

3. (a) D = (5,2) (b) E may have coordinates such as (4,1) or (3,0) or any other point along the line CD OR (2,3), (1,2), (0,1) or any other point along the line BA

4. AH: x = 10 DE: x = -10 BC: y = 10 EA: y = ½ x or 2y = x AB: x + y = 15

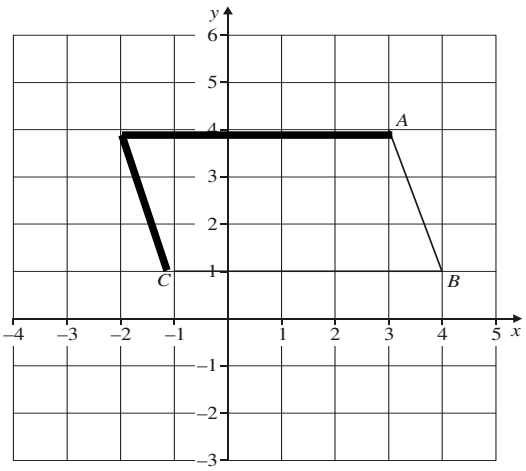
5. AD = 12cm; DC = 6cm;
As AC=CD=AD (12cm) triangle ACD is equilateral
So y = 60° x = 30°

6.



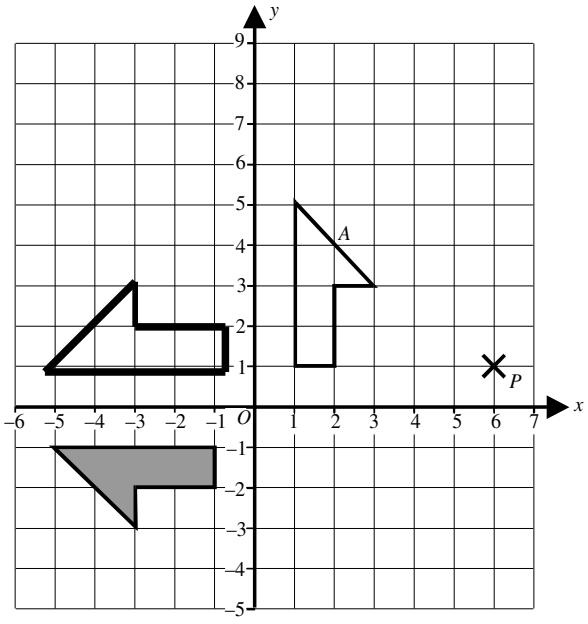
Answers to Past Paper Questions

1. (a) (3,4) (b) (-1, 1) (c) (i) below (c) (ii) (-2, 4)



2. (a) A reflection about the line y - -x

(b)

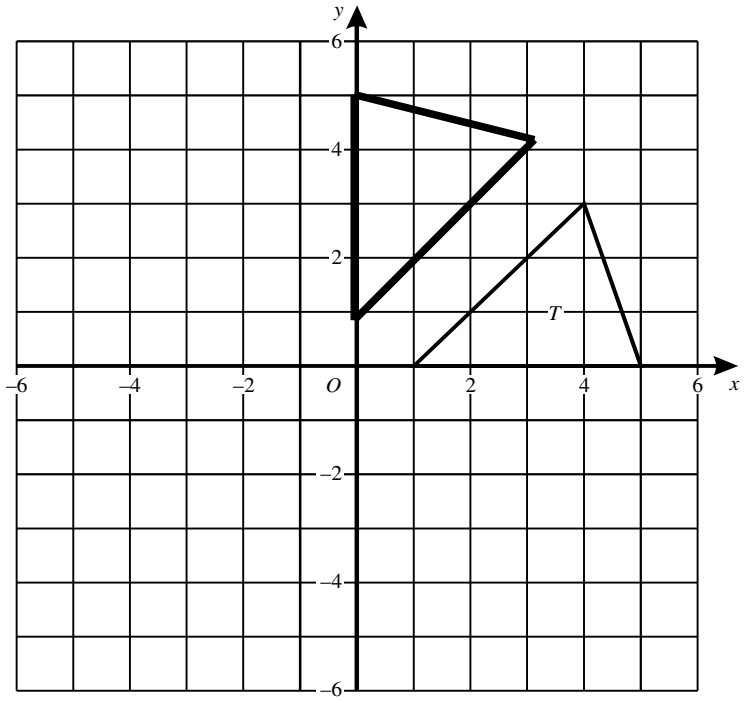


3. (a) reflection in the line y = 0 or x axis
   (b) reflection in the line y = x

4. (a) Reflection in the line y -1
   (b) Rotation 90 clockwise about the centre (-1, -1)

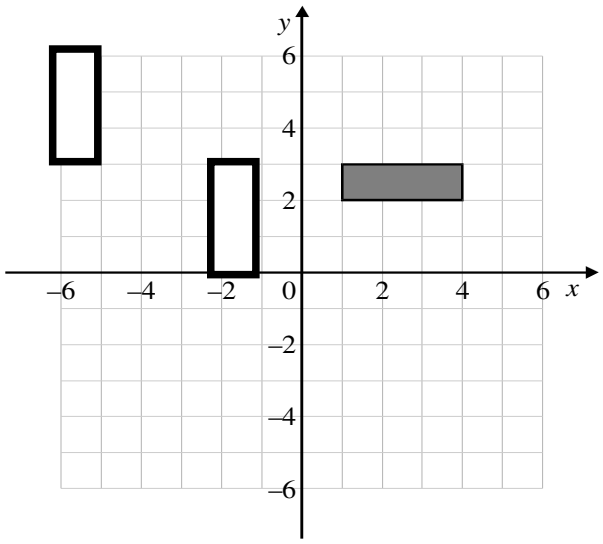
5. (-4, -1)

6.

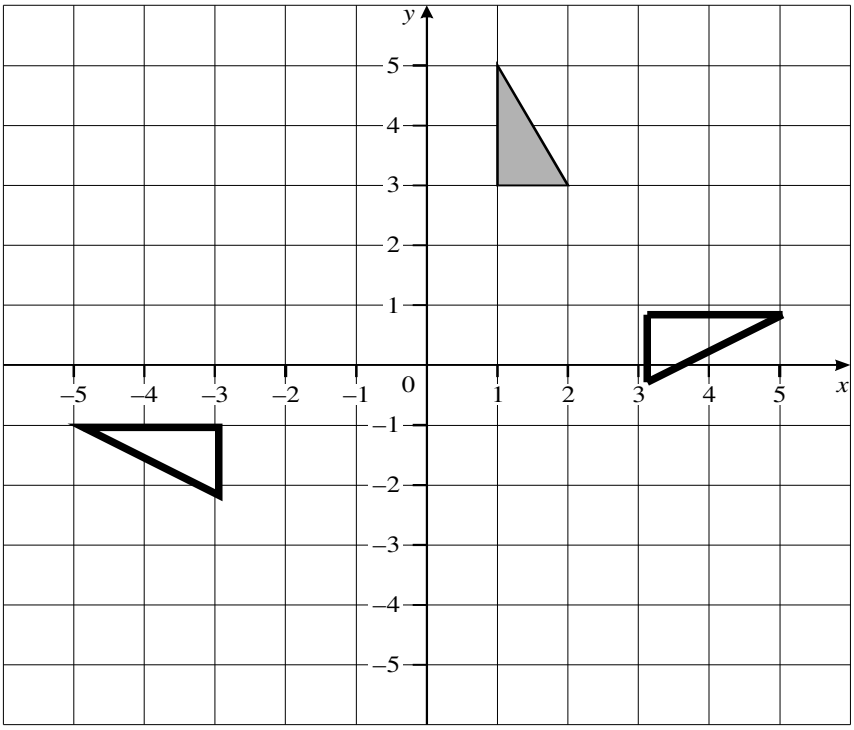


7 (a) Reflection about the line y = -x

(b)

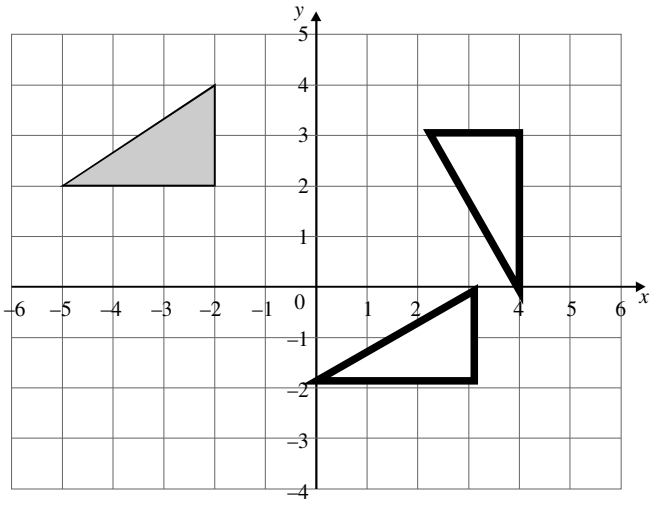


8.

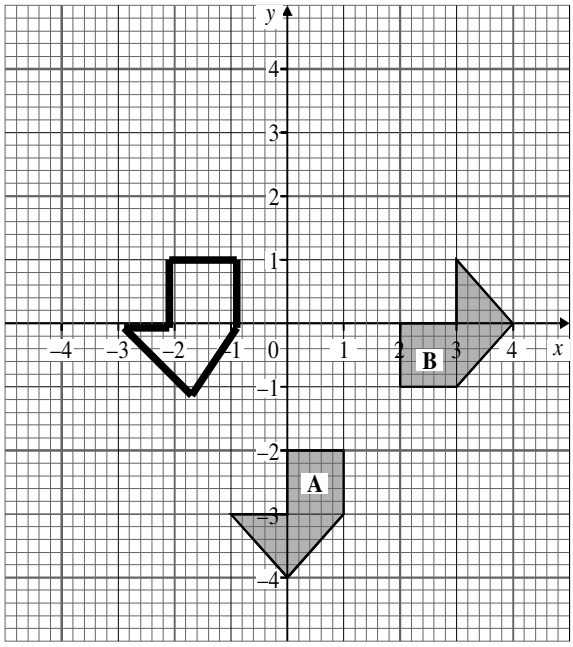


(b) 180 rotation about the centre (0, -1)

9.



10.



END OF ANSWERS

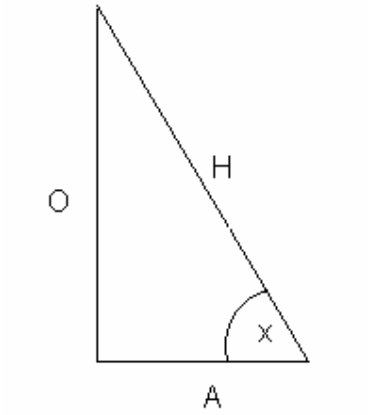
30-4-10 Shape and Space
DRAFT

Topic: Trigonometry

You need to be able to:

- Use the Trigonometry ratios to calculate the missing side in right angled triangles given a side and an angle
- Use the Trigonometry ratios to calculate a missing angle in right angled triangles given two sides
- Use Trigonometry in real-life situations

You will need to think about:



Ratios:

$\sin x = \frac{O}{H}$ $\cos x = \frac{A}{H}$

$\tan x = \frac{O}{A}$

You may be asked to use these facts in real life situations.

Remember:

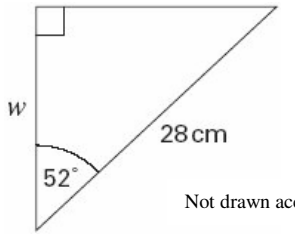
- If there is not a triangle drawn for you, draw one yourself!
- If the marked angle moves, so do the Opposite and Adjacent sides – but not the Hypotenuse!

Hint:

Many people use the phrase SOH CAH TOA to help them remember the ratios – can you work out what it means?

Quick Questions

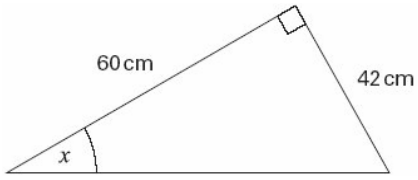
1. (a) Calculate the length *w*



Not drawn accurately

w = ... cm
2 marks

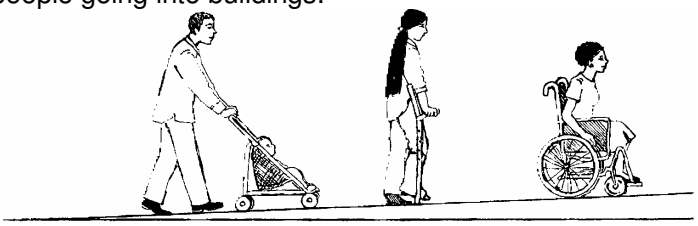
(b) Calculate the size of angle *x*



Not drawn accurately

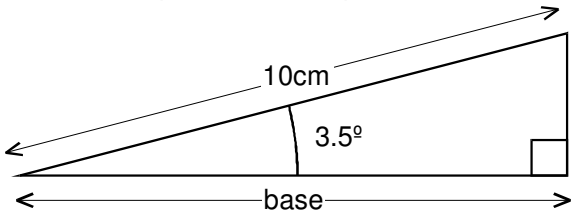
x = ...°
2 marks

2. Ramps help people going into buildings.



A ramp that is **10m long** must not have a **height** greater than **0.83m**.

(a) Here are the plans for a ramp:



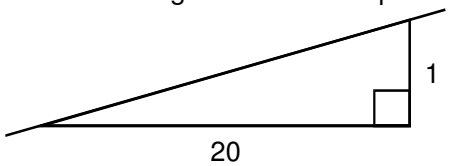
NOT TO SCALE

How long is the base of this ramp?
You **must** show your calculations.

..... m

2 marks

(b) The recommended gradient of a ramp is 1 in 20



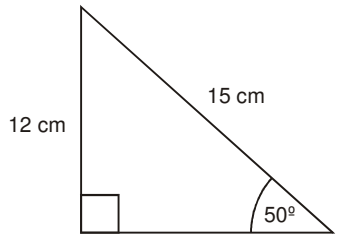
NOT TO SCALE

What angle gives the recommended gradient?
You **must** show your calculations.

..... °

2 marks
Total 4 marks

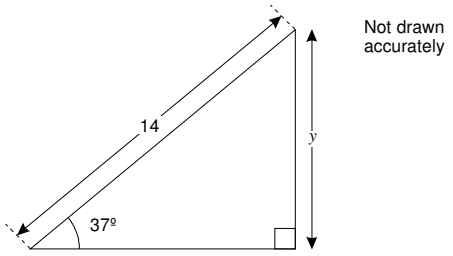
3. **Is it possible** to have a triangle with the angles and lengths shown below?
You must show calculations then tick (✓) Yes or No.



Decision: ☐ Yes ☐ No

2 marks

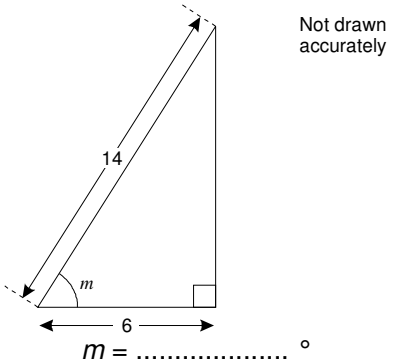
4. (a) Calculate the value of *y*
Show your working.



y =.....

2 marks

b) Calculate the value of angle *m*
Show your working.

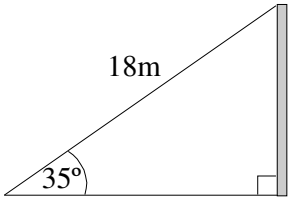


m = ... °

2 marks

Past Examination Questions *(From AQA GCSE papers)*

1. A wire 18 m long runs from the top of a pole to the ground as shown in the diagram.
The wire makes an angle of 35º with the ground.

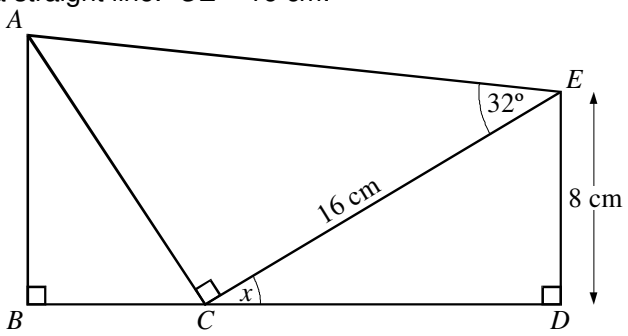


Calculate the height of the pole.
Give your answer to a suitable degree of accuracy.

.....
.....
.....

(4)

2. *BCD* is a straight line. *CE* = 16 cm.



The diagram is not drawn to scale.

(a) Calculate the size of the angle marked *x*.

.....
.....
.....

(3)

(b) Calculate the length of *AC*.

.....
.....
.....

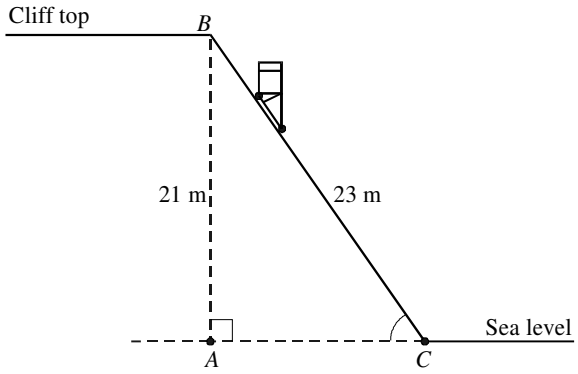
(3)

(c) Use your answers to parts (a) and (b) to calculate the length of *BC*.

.....
.....
.....

(3)

3. A lift at the seaside takes people from sea level to the top of a cliff, as shown.



Not to scale

From sea level to the top of the cliff, the lift travels 23 m and rises a height of 21 m.

(a) Calculate the distance *AC*.

.....

.....

Answer..... m

(3)

(b) Calculate angle *BCA*.

.....

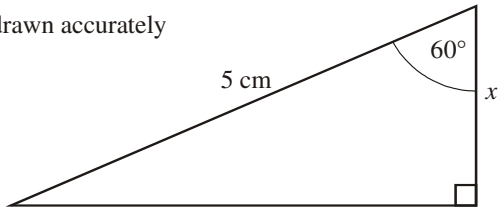
.....

Answer..... degrees

(3)

4. Use the information in the table to calculate the value of *x*.

Not drawn accurately



sin 60° = 0.866
cos 60° = 0.5
tan 60° = 1.732

.....

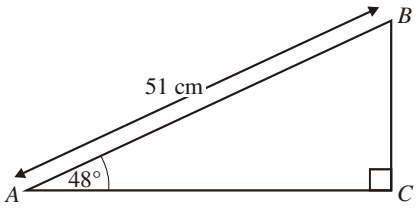
.....

Answer ..... cm

(3)

(Total 3 marks)

5. *ABC* is a right-angled triangle.
AB = 51 cm
Angle *CAB* = 48°



Not to scale

Find the length of *BC*.
Give your answer to a suitable degree of accuracy.

.....

.....

Answer ..... cm

(Total 4 marks)

Mark Scheme

Quick Questions

1. (a) 28 cos 52 = 17 or 17.2(…), with no evidence of accurate or scale drawing — 2

(b) $\tan^{-1}\frac{42}{60}$ = 35 or 34.9(…), with no evidence of accurate or scale drawing — 2

[4]

2. (a) 10 x cos 3.5 = 9.98(....) — 2

(b) $\tan^{-1}\frac{1}{20}$ = 2.86 — 2

[4]

3. Indicates No with reason:
Eg: The angle should be 53
The hypotenuse should be 15.6… — 2

4. (a) 14 x sin 37 = 8.4(...) — 2

(b) $\cos^{-1}\frac{6}{14}$ = 64.6(...) — 2

[4]

Examination Questions

1. 18 x sin 35° — M3
10, 10.3, 10.32 *3 dp or more A0* — A1

[4]

2. (a) sin-1 $\frac{8}{16}$ = 30° — M2 A1
(b) 16 x tan 32° = 10 cm — M2 A1
(c) 180° – 90° – 30° = 60° — B1
BC = cos 60° 10 — M1
BC = 5 — A1

[9]

3. (a) $\sqrt{(23^2 - 21^2)}$ — M2
= 9.3(8 ...) or 9.4 — A1
(b) sin-1 $\frac{21}{24}$ = 65.9° — M2 A1

[6]

4. 5 x cos 60 — M1
= 5 x 0.5 — M1
= 2.5 — A1

[3]

5. 51 x sin 48° — M2
= 37.9(...) — A1
≈ 37.9 or 38 *accuracy mark* — B1

[4]

30-4-10 Shape and Space
DRAFT

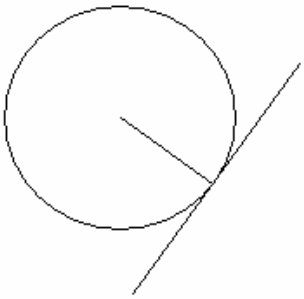
Enabling world-class education

Topic: Circle Theorems

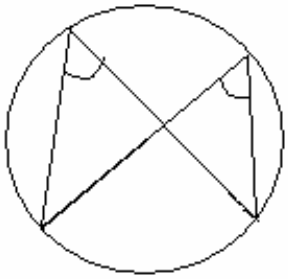
You need to be able to:

- Recognize and use the circle theorems to find angles and explain your thinking

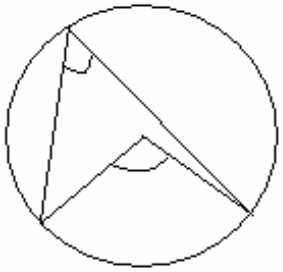
These are the theorems you need to know:



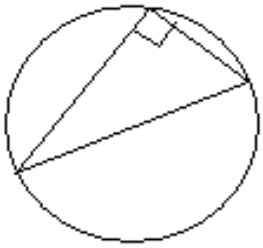
A radius and a tangent always meet at a right angle



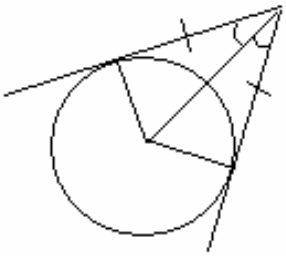
Two angles standing on the same arc are equal



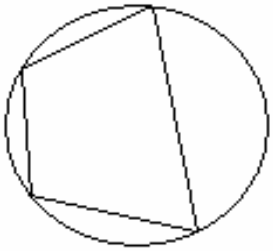
The angle at the centre of a circle is twice the angle at the circumference when they are standing on the same arc.



The angle at the circumference from the diameter is a right angle



From a point outside the circle, the length of the two tangents is equal and the line from the point to the centre bisects the angle



Opposite angles in a cyclic quadrilateral add up to 180°.

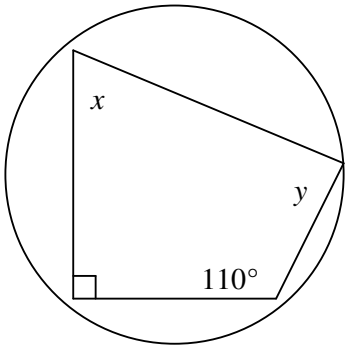
Remember: If there is not a diagram drawn for you, sketch one yourself!

Hint: If you can't see the answer immediately, write all of the angles you do know in the diagram!

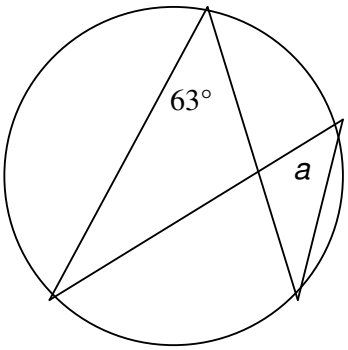
Quick Questions

(These diagrams are not to scale)

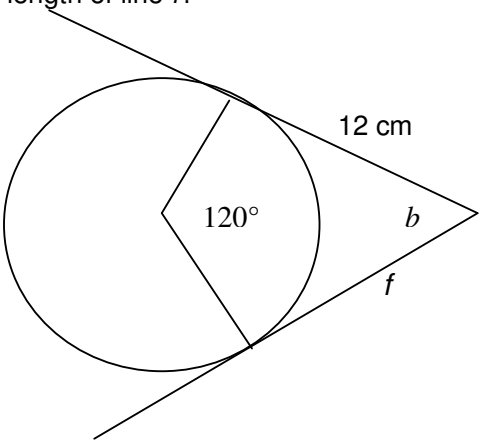
1. Find the values of *x* and *y*.



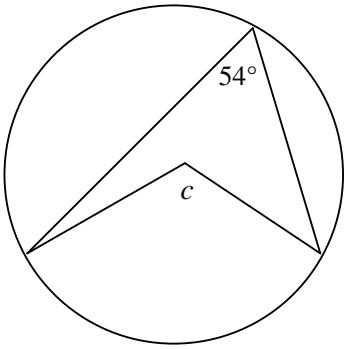
2. What number does *a* represent?



3. What is the size of angle *b* and the length of line *f*?



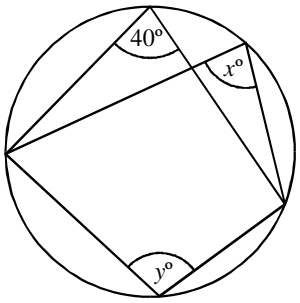
4. Find the value of angle *c*.



5. Prove that the angle at the circumference standing on the diameter is a right angle. Use another circle theorem to help you.

Past Examination Questions *(From AQA GCSE papers)*

1.



Not drawn accurately

(i) Write down the value of *x*.

Answer ..... degrees

(1)

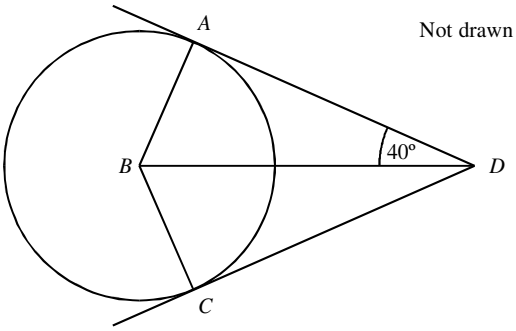
(ii) Calculate the value of *y*.

.....

Answer ..... degrees

(1)

(b) *A* and *C* are points on the circumference of a circle centre *B*. *AD* and *CD* are tangents.
Angle *ADB* = 40°.



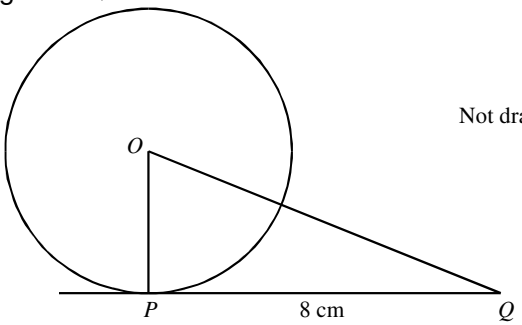
Not drawn accurately

Explain why angle *ABC* is 100°.

.....

(2)

(c) *P* is a point on the circumference of a circle with centre *O*.
PQ is a tangent of length 8 cm.
The area of triangle *OPQ* is 24 cm^2.



Not drawn accurately

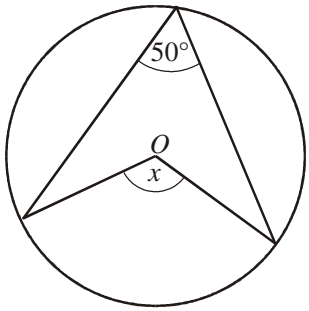
Calculate the area of the circle.
Give your answer in terms of π.

.....

Answer ..... cm^2

(3)

2. (a) The diagram shows a circle with centre *O*.



Not drawn accurately

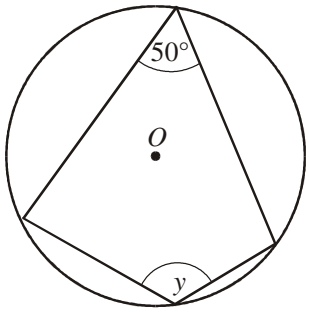
Work out the size of the angle marked *x*.

.....

Answer ..... degrees

(1)

(b) The diagram shows a different circle with centre *O*.



Not drawn accurately

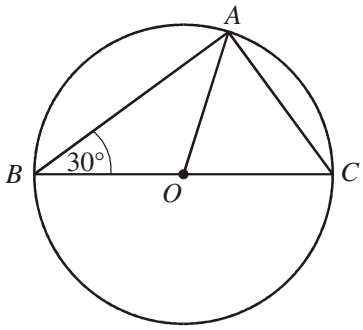
Work out the size of the angle marked *y*.

.....

Answer ..... degrees

(1)

(c) *A*, *B* and *C* are points on the circumference of a circle with centre *O*.
BOC is a diameter of the circle.
Angle *ABC* = 30°



Not drawn accurately

Explain why triangle *OAC* is equilateral.

.....

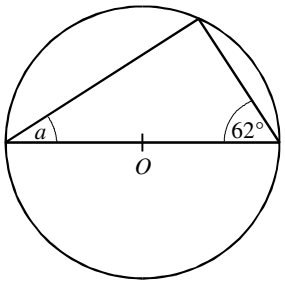
.....

.....

(3)

(Total 5 marks)

3. In the diagram, *O* is the centre of the circle.



Not drawn accurately

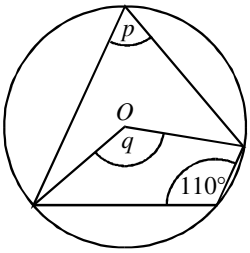
Calculate the value of *a*.

.....

Answer ..... degrees

(2)

4. *O* is the centre of the circle.



not drawn accurately

(a) Calculate the value of angle *p*.

.....

Answer *p* = ... degrees

Reason .....

(2)

(b) Calculate the value of angle *q*.
Give a reason for your answer.

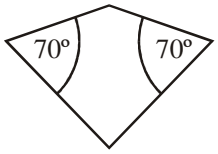
Answer *q* = ... degrees

Reason .....

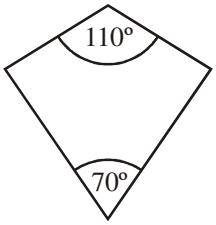
(2)

5. Which **one** of the following kites is a cyclic quadrilateral? Give a reason for your answer.

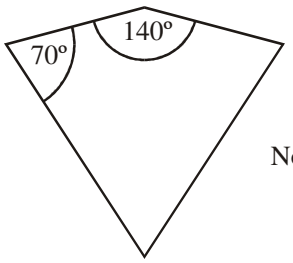
Kite 1



Kite 2



Kite 3



Not drawn accurately

Answer .....

Reason .....

(Total 2 marks)

Answers

Quick Questions

1. $x = 70°$, $y = 90°$ *because opposite angles in a cyclic quadrilateral add up to 180°*
2. $a = 63°$ *because angles at the circumference standing on the same arc are equal*
3. $b = 60°$ *because the radius meets the tangent at right angles so b = 360-120-90-90*
   $f = 12$ cm *because the length of two tangents from the same point to the circle is equal*
4. $c = 108°$ *because the angle at the centre is twice the angle at the circumference when standing on the same arc.*
5. *Where t*he diameter passes through the centre of the circle, the angle is 180°. As the angle at the circumference is standing on the same arc, it must be half and half of 180° is 90°.

Past Examination Questions

1.	(a)	(i)	40	B1
		(ii)	140	B1
	(b)	*eg*	in triangle, radius meets tangent at 90° so ABD 180-90-40=50°	
			ABC is twice ABD so ABC is 100°	B2
	(c)		$24 \div 8 \times 2 = 6$ which is the radius	M1
			$\pi \times 6^2$	M1
			$=36\pi$	M1

[7]

2.	(a)	100°	B1
	(b)	130°	B1
	(c)	OAB = 30°	M1
		BAC = 90° so OAC = 60°	M1
		BCA = 60° so triangle is equilateral	A1

[5]

3.	(a)	180 – 190 – 62 or 90 – 62	M1
		28°	A1

4.	(i)	70°	B1
		(opposite angles of) cyclic quadrilateral	B1
	(ii)	140°	B1
		angle at centre is twice angle at circumference	B1

[4]

5.	Kite 2 and reason	B2
	e.g. 110 + 70 = 180	
	sum of (opposite) angles = 180	

[2]

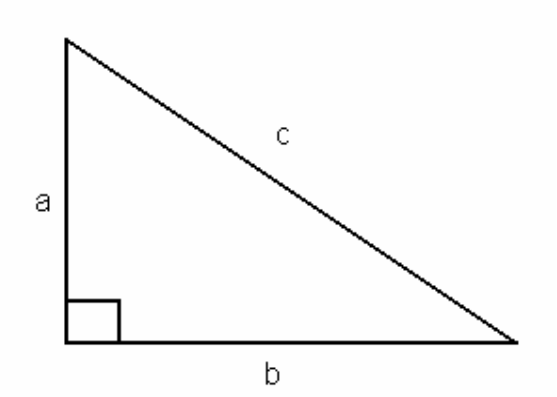
30-4-10 Shape and Space
DRAFT

Topic: Pythagoras' Theorem

You need to be able to:

- Use Pythagoras's Theorem to calculate the missing side in right angled triangles given two sides
- Use Pythagoras' Theorem in real-life situations

You will need to think about:



The Theorem states:

For a right angled triangle:

$c^2 = a^2 + b^2$

You may be asked to use these facts in real life situations.

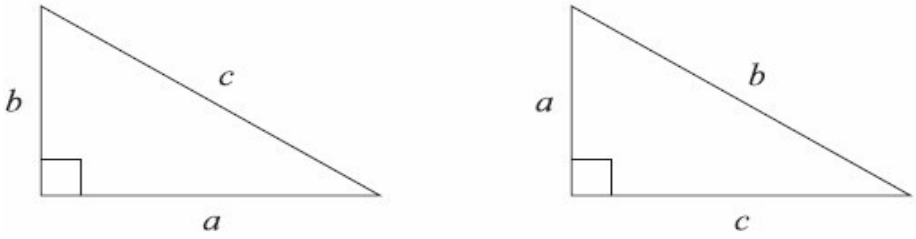
Remember: If there is not a triangle drawn for you, draw one yourself!

Hint:

- Looking for the hypotenuse (long side) – square the other two and **add** before square rooting
- Looking for a short side – square the other two and **subtract** before square rooting

Quick Questions

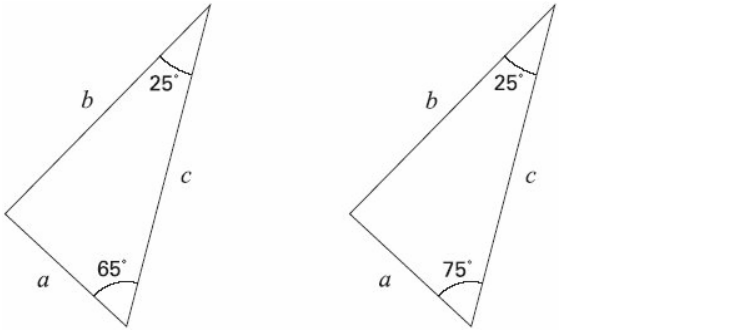
1. (a) In which triangle below does $a^2 + b^2 = c^2$?
Tick (✓) the correct triangle.



For the **other** triangle, write an equation linking a, b and c

1 mark

(b) In which triangle below does $a^2 + b^2 = c^2$?
Tick (✓) the correct triangle.

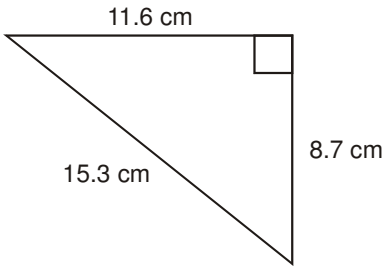


Not drawn accurately

For the **other** triangle, explain why $a^2 + b^2$ does not equal c^2

1 mark

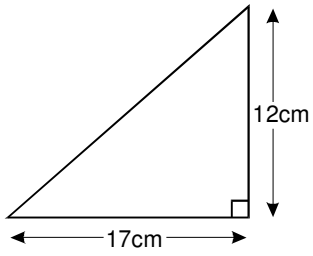
2. **Is it possible** to have the triangle shown below?
Make sure you show calculations then tick (✓) Yes or No.



Decision: ☐ Yes ☐ No

2 marks

3. (a) Calculate the length of the unknown side of this right-angled triangle.
Show your working.

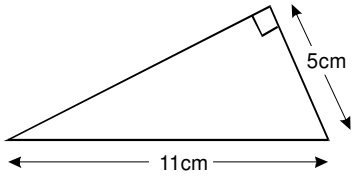


Not drawn accurately

..... cm

2 marks

(b) Calculate the length of the unknown side of the right-angled triangle below.
Show your working.

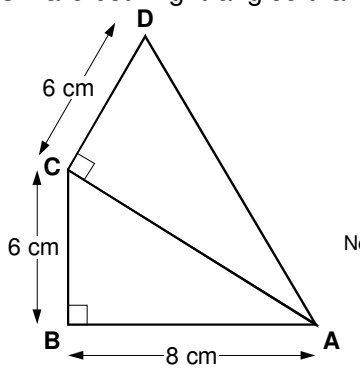


Not drawn accurately

..... cm

2 marks

4. ABC and ACD are both right-angled triangles.



Not drawn accurately

(a) Explain why the length of AC is 10 cm.

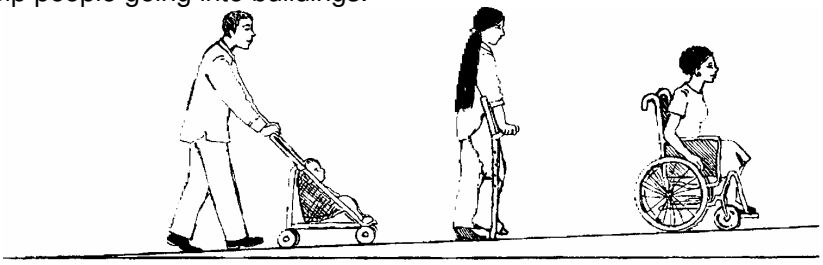
1 mark

(b) Calculate the length of AD
Show your working.

..... cm

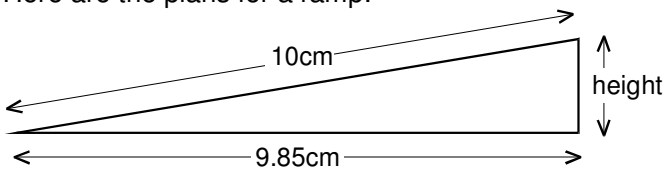
2 marks

5. Ramps help people going into buildings.



A ramp that is **10m long** must not have a **height** greater than **0.83m**.

(a) Here are the plans for a ramp:



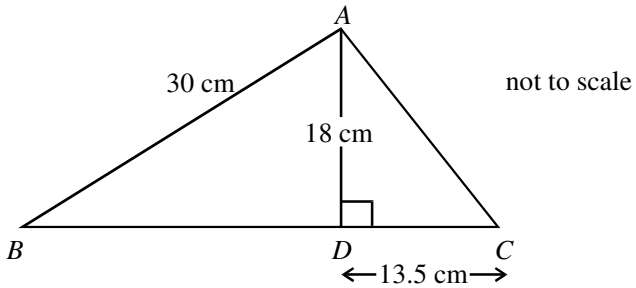
NOT TO SCALE

Is this ramp too high?
You **must** show calculations to explain your answer.

2 marks

Past Examination Questions *(From AQA GCSE Papers)*

1.



BDC is a straight line.

Use Pythagoras' theorem

(a) to calculate the length of *AC*,

.....

.....

(2)

(b) to calculate the length of *BD*,

.....

.....

(2)

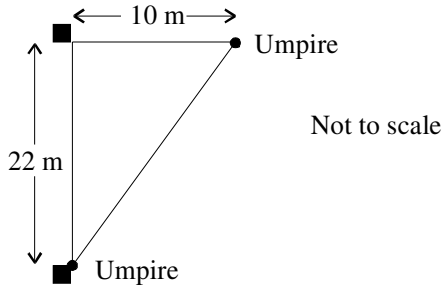
(c) to show that angle *BAC* is a right angle.

.....

.....

(2)

2. In a game two umpires stand on a pitch as shown.



How far apart are the two umpires?

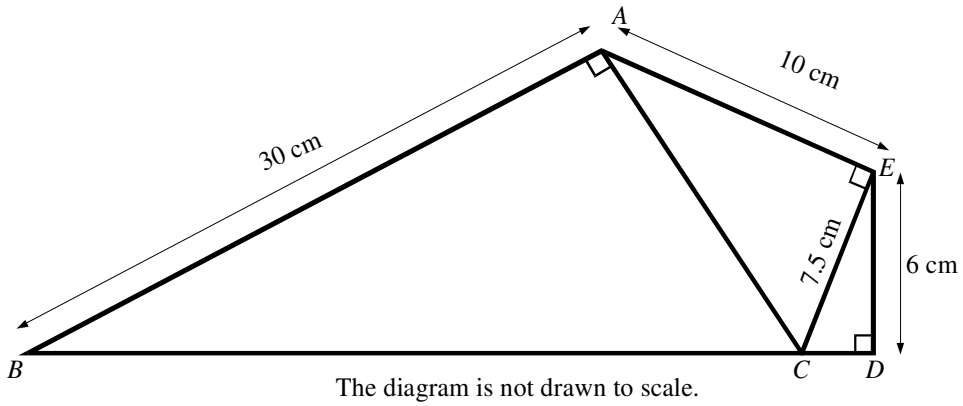
.....

.....

.....

(3)

3. $CE = 7.5$ cm



The diagram is not drawn to scale.

Use Pythagoras' theorem to calculate

(a) the length of *CD*,

.....

.....

(2)

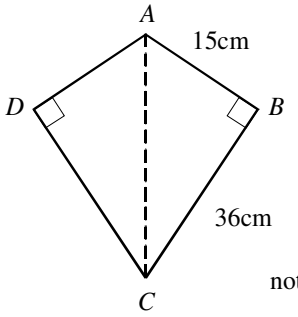
(b) the length of *CB*.

.....

.....

(3)

4. The diagram shows a kite *ABCD*.



not drawn to scale

(a) Calculate the area of the kite *ABCD*.
Show all your working.
Remember to state the units in your answer.

.....

.....

(4)

(b) Calculate the length of *AC*.
Remember to state the units in your answer.

.....

.....

(4)

Mark Scheme

Quick Questions

1. (a) First triangle ticked and correct equation for second: $a^2 + c^2 = b^2$ 1
($b^2 - a^2 = c^2$ or $b^2 - c^2 = a^2$ or $b = \sqrt{a^2 + c^2}$)
(b) First triangle ticked and reason: 25 + 75 = 100 so the other angle must be 80 so there are no right angles and you can only use Pythagoras on right angled triangles 1
[2]

2. Indicates No and gives a correct justification eg $11.6^2 + 8.7^2 \neq 15.3^2$ 2
[2]

3. (a) $\sqrt{(17^2 + 12^2)}$
= 20.8(...) or $\sqrt{433}$ 2
(b) $\sqrt{(11^2 - 5^2)}$
= 9.8(0) or 9.79(...) or $\sqrt{96}$ 2
[4]

4. (a) eg $6^2 + 8^2 = 10^2$ 1
(b) $\sqrt{(10^2 + 6^2)}$
= $\sqrt{136}$ or 11.7 or 11.6(..) 2
[3]

5. $\sqrt{(10^2 - 9.85^2)} = 1.725\ldots$ which is too high 2
[2]

Examination Questions

1. (a) $AC^2 = 18^2 + (13.5)^2$ M1
AC = 22.5 A1
(b) $BD^2 + 18^2 = 30^2$ M1
BD = 24 A1
(c) $30^2 + (22.5)^2 = (24 + 13.5)^2$ or finding angles BAD and DAC M1
1406.25 = 1406.25 53.1° + 36.9° = 90° A1
[6]

2. $10^2 + 22^2$ M1
$\sqrt{}584$ = 24.2 or 24 M1 A1
[3]

3. (a) $CD^2 + 6^2 = 7.5$ M1
CD = 4.5 A1
(b) $CA^2 = 7.5^2 + 10^2$ M1
$CB^2 = 30^2 +$ (candidate's CA)^2 M1
CB = 32.5 A1
[5]

4. (a) ½ × 36 × 15 = 270 M1 A1
270 × 2 = 540 cm² M1 A1
(b) $15^2 + 36^2$ or 1521 M1
$\sqrt{1521}$ = 39 cm M1 A1
both units correct B1
[8]